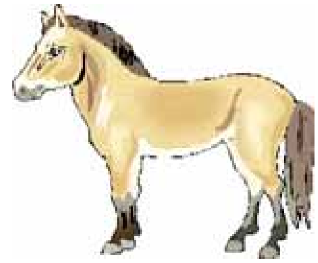
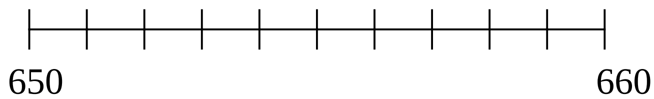
A Question of Numbers

This problem gives you the chance to:
- show you can compare and order numbers



Holly's class is learning about big numbers.

1. Here is part of the class number line.



Put an **X** on the number line, on the place that is halfway between 650 and 660.

What is the number that should be there? _______________

2. Holly knows a pony weighs between 365 pounds and 425 pounds.

   Write a possible number for the weight of a pony. _______________ pounds

3. The activity center swimming pool holds between 1,875 gallons and 1,940 gallons of water.

   Write a possible number for the amount of water the swimming pool holds. _______________ gallons

4. A school computer could cost between \$2,950 and \$3,055.

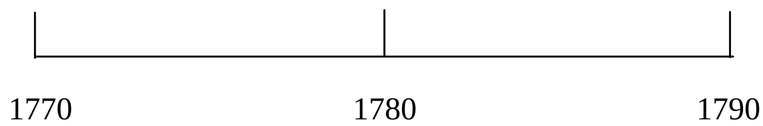
   Give three possible prices for the computer.

   \$ _______________ \$ _______________ \$ _______________

5. The American Revolution started in 1775 and finished in 1783.

In which year was it halfway through? _________________

Show how you know using this number line.



6. Holly's teacher says the school library has 1000 books.
Holly thinks that the library may have 40 more books.

What is the greatest number of books that the library could have? __________

Show how you figured this out.

Tom thinks that the library may have 40 fewer books than 1000.
What is the smallest number of books that the library could have? ____________

Show how you figured this out.

8

Task 5: A Question of Numbers Rubric

• • The core elements of performance required by this task are: • • show you can compare and order numbers • Based on these, credit for specific aspects of performance should be assigned as follows		points	section points
1.	Gives correct answer: **655**	1	
	Draws an **X** in the correct place on the number line.	1	2
2.	Gives correct answer such as: 380 pounds.	1	1
3	Gives correct answer: such as 1,920 gallons.	1	1
4.	Gives correct answers such as: \$2955 \$3000 \$3053	1	1
5	Gives correct answer: **1779 and**		
	Marks the correct year on the number line.	1	1
6.	Gives correct answer: **1040 and**		
	Shows 1000 + 40	1	
	Gives correct answer: **960 and**		
	Shows 1000 - 40	1	2
	Total Points		**8**

A Question of Numbers

Work the task and look at the rubric. What do are the big mathematical ideas being assessed by this task?

In part 1, students were asked to use a number line to find a number between 650 and 660. How many of your students put:

655 mark on # line	655 no mark on # line	655.5	656	651	1310	Other

Think about each of these mistakes. How are they showing different misunderstandings about number and place value?

To work parts 2,3,and 4 students had to think about several things. What does between mean? Should I do a computation? How do I make a number larger or smaller? In looking at place value, Liping Ma and Kathy Richards talk about numbers going around ten, numbers going around hundreds, and numbers going around a thousand. Look carefully at how your students thought about these ideas. How many of your students:

Correct Answers	Misunderstood "between" *Put in the end values*	Added or subtracted the two numbers	Values for part 4 were too high or too low	Other

Now look at work for part 5. Have your students worked with number lines? If they haven't worked with number lines, what other strategies could they use to make sense of the problem?

Look at student work.

1179	1779, but no use of number line	1778	1780	1775/1790	Other

Did students have trouble drawing the scale?

Did students have trouble counting marks instead of spaces?

What other misconceptions did you see?

What are some ways that you could incorporate use of the number line in warm ups? Problem-solving? Number talks? How does the number line help to develop a deeper understanding of place value? Of number sense? Of how the base-ten system is built?

Finally look at work for part 6. Did students know to add and subtract? Did they pick 1000 for most and 40 for least? When subtracting what types of place-value understanding errors did you see?

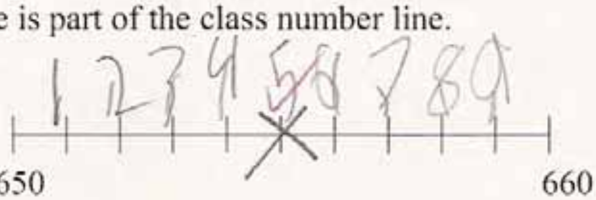
Looking at Student Work on A Question of Numbers

Student A is able to use the number line to find numbers within a given range or to divide a range halfway. The student understands the mathematical use of the word between and uses landmark or friendly numbers to name values between. Notice how Student A puts all the values on the number line, identifies the section being considered, then labels the midpoint.

Student A

Holly's class is learning about big numbers.

1. Here is part of the class number line.



+2

Put an **X** on the number line, on the place that is halfway between 650 and 660.

What is the number that should be there? 655

+1

2. Holly knows a pony weighs between 365 pounds and 425 pounds.

Write a possible number for the weight of a pony. 400 pounds

3. The activity center swimming pool holds between 1,875 gallons and 1,940 gallons of water.

Write a possible number for the amount of water the swimming pool holds. 1,900 gallons +1

4. A school computer could cost between \$2,950 and \$3,055. +1

Give three possible prices for the computer.

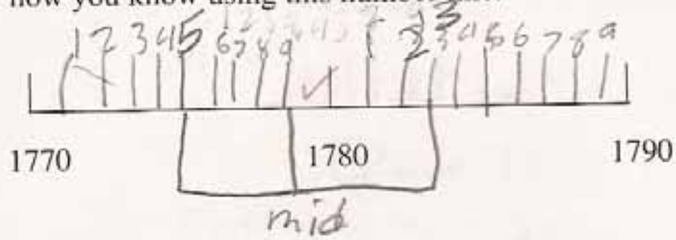
\$ 2,975 \$ 3,000 \$ 3,025

Student A, part 2

5. The American Revolution started in 1775 and finished in 1783.
In which year was it halfway through? 1779

+1

Show how you know using this number line.



6. Holly's teacher says the school library has 1000 books.
Holly thinks that the library may have 40 more books.
What is the greatest number of books that the library could have? 1040
Show how you figured this out.

+1

$$\begin{array}{r} 1000 \\ +\ \ 40 \\ \hline 1{,}040 \end{array}$$

Tom thinks that the library may have 40 fewer books than 1000.
What is the smallest number of books that the library could have? 960
Show how you figured this out.

+1

$$\begin{array}{r} 1000 \\ -\ \ 40 \\ \hline 960 \end{array}$$

3

8

8

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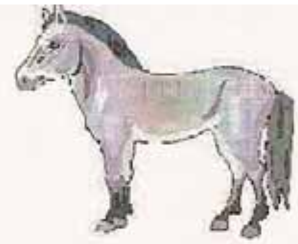
A Question of Numbers Test 3

Student B uses the number line as a tool. See the marking off of equal distances to locate the midpoint in part 1. The student sees the number line as a series of numbers between two end points, but has not quite mastered how to scale in all the values or measure in equal units. (See part 5. The student is trying to count backwards to the middle, but makes a stumble. The student has a very clear, clean explanation for finding the most and least books.

Student B



Holly's class is learning about big numbers.

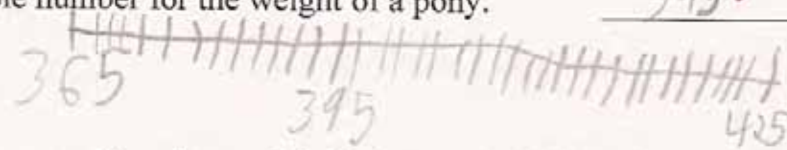
1. Here is part of the class number line.

Put an **X** on the number line, on the place that is halfway between 650 and 660. 1

What is the number that should be there? 655 ✓ 1

2. Holly knows a pony weighs between 365 pounds and 425 pounds.

Write a possible number for the weight of a pony. 395 ✓ pounds 1



3. The activity center swimming pool holds between 1,875 gallons and 1,940 gallons of water.

Write a possible number for the amount of water the swimming pool holds. 1,897 ✓ gallons 1

4. A school computer could cost between \$2,950 and \$3,055.

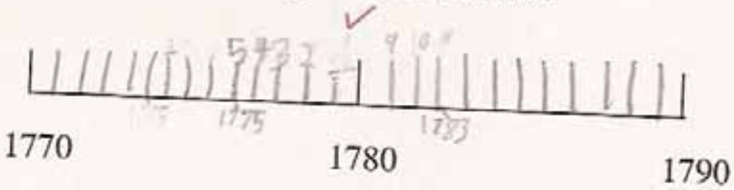
Give three possible prices for the computer.

\$ 2,975 ✓ \$ 2,993 ✓ \$ 3,042 ✓ 1

Student B, part 2

5. The American Revolution started in 1775 and finished in 1783.
   In which year was it halfway through? 1779 ✓

   Show how you know using this number line.



6. Holly's teacher says the school library has 1000 books.
   Holly thinks that the library may have 40 more books.
   What is the greatest number of books that the library could have? 1,040 ✓
   Show how you figured this out.

   Because it <u>may</u> have 40 more books. So add 40 to 1,000. You get 1,040 ✓

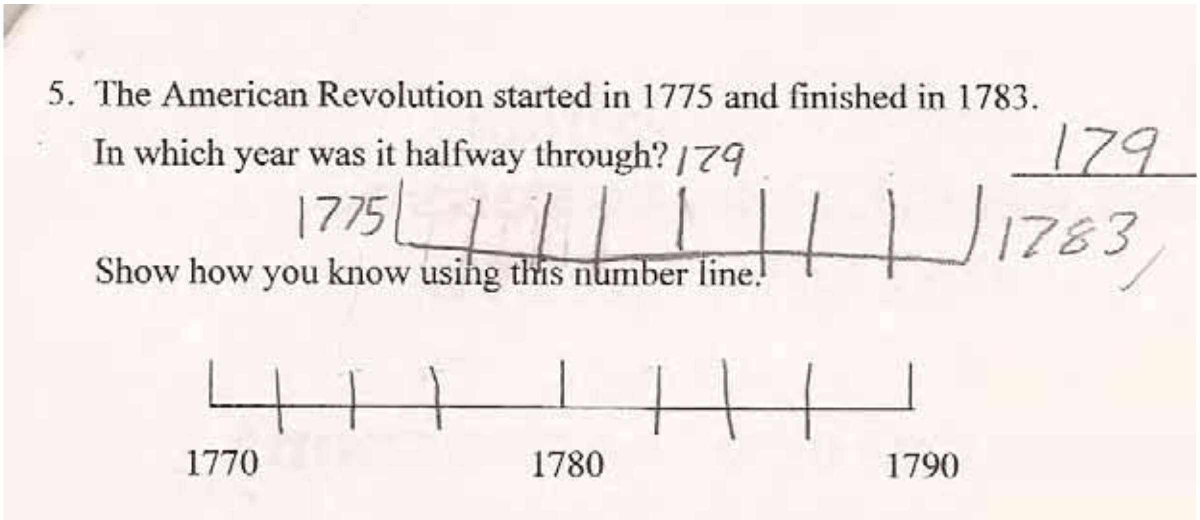
   Tom thinks that the library may have 40 fewer books than 1000.
   What is the smallest number of books that the library could have? 960 ✓
   Show how you figured this out.

   Because it <u>may</u> have 40 less than 1,000. So subtract 40 from 1,000. You get 960. ✓

   8 8

This idea of measuring off equal units or scaling the number line was difficult for students. Notice Student C attempted the idea and then simplified the number line to just the relevant numbers. Student D was able to think about the locations for starting and ending numbers and then put in the appropriate midpoint without counting every number. Student E combines measuring equal units, with marking off the starting, ending and midpoints.

Student C


5. The American Revolution started in 1775 and finished in 1783.

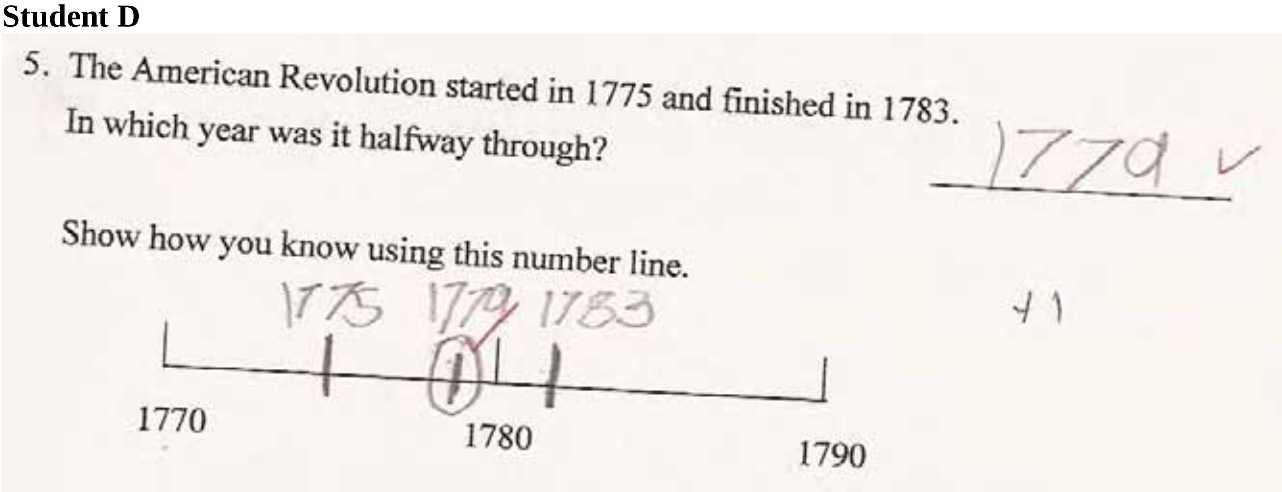
In which year was it halfway through? 179   179

1775   1783

Show how you know using this number line.

1770   1780   1790

Student D


5. The American Revolution started in 1775 and finished in 1783.

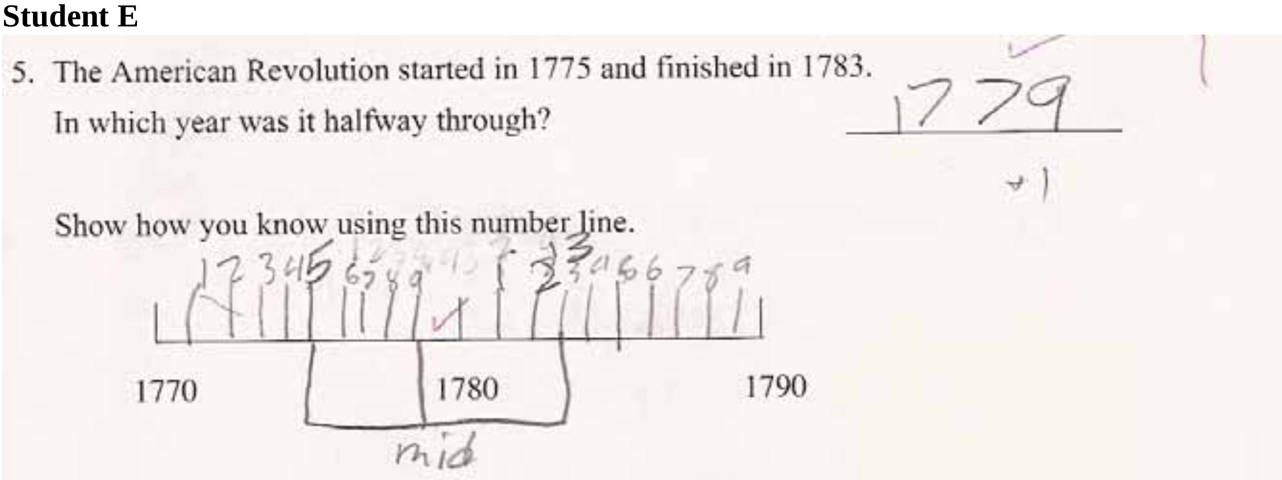
In which year was it halfway through? 1779

Show how you know using this number line.

1775 1779 1783   +1

1770   1780   1790

Student E


5. The American Revolution started in 1775 and finished in 1783.

In which year was it halfway through? 1779

+1

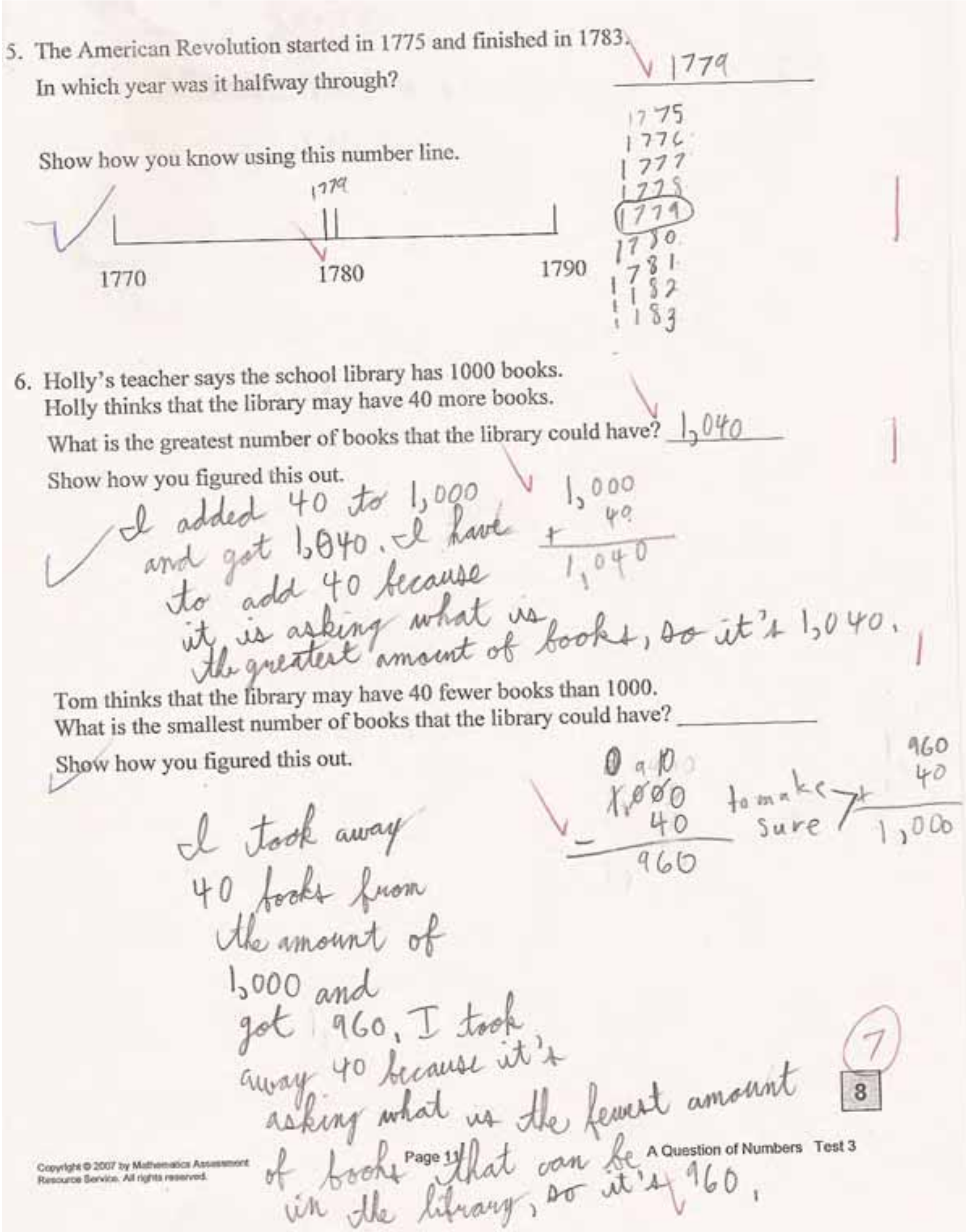
Show how you know using this number line.

1770   1780   1790

mid

Student F may or may not have had experience with a number line, but is able to use other strategies to find the midpoint. Then Student F is able to estimate the location of that value on the number line. Notice the full explanation for finding the values in part 6.

Student F



5. The American Revolution started in 1775 and finished in 1783.
In which year was it halfway through? 1779

Show how you know using this number line.

1770 — 1779 — 1780 — 1790

1775
1776
1777
1778
1779
1780
1781
1782
1783

6. Holly's teacher says the school library has 1000 books.
Holly thinks that the library may have 40 more books.
What is the greatest number of books that the library could have? 1,040

Show how you figured this out.

I added 40 to 1,000 and got 1,040. I have to add 40 because it is asking what is the greatest amount of books, so it's 1,040.

$1,000 + 40 = 1,040$

Tom thinks that the library may have 40 fewer books than 1000.
What is the smallest number of books that the library could have?

Show how you figured this out.

I took away 40 books from the amount of 1,000 and got 960. I took away 40 because it's asking what is the fewest amount of books that can be in the library, so it's 960.

$1,000 - 40 = 960$

to make sure $960 + 40 = 1,000$

7

8

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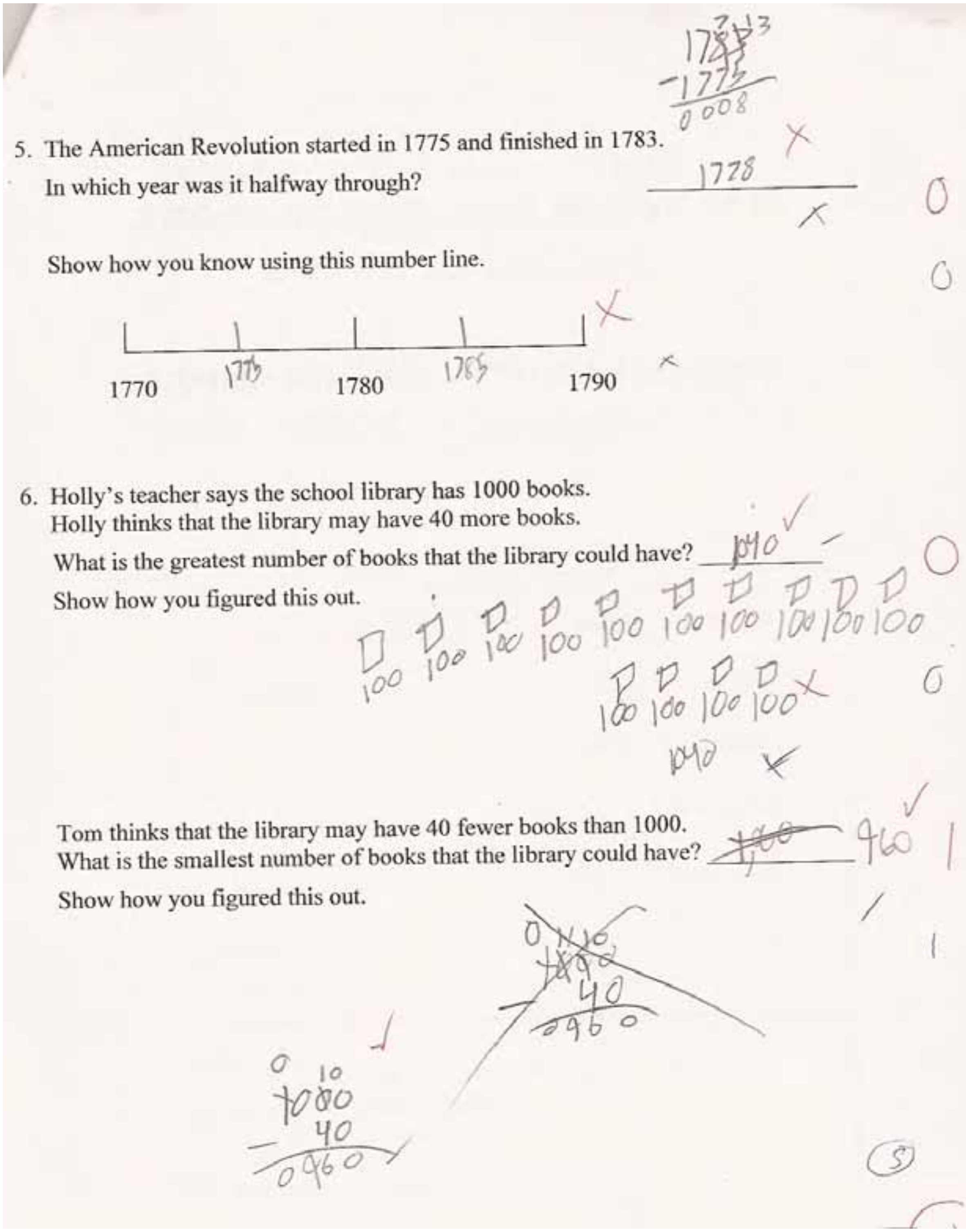
A Question of Numbers Test 3

Student G tries to use subtraction to find the midpoint. But does not use that information to find the midpoint. *How could the "8" be used to find the midpoint? What would be the next step(s)?* Notice how the student uses place value units to find the total number of books in the library.

Student G



1783
−1775
0008

5. The American Revolution started in 1775 and finished in 1783.
In which year was it halfway through? 1728

Show how you know using this number line.

1770 1775 1780 1785 1790

6. Holly's teacher says the school library has 1000 books.
Holly thinks that the library may have 40 more books.
What is the greatest number of books that the library could have? 1040
Show how you figured this out.

100 100 100 100 100 100 100 100 100 100
100 100 100 100
1040

Tom thinks that the library may have 40 fewer books than 1000.
What is the smallest number of books that the library could have? ~~1,00~~ 960
Show how you figured this out.

1000
− 40
0960

Student H is able to measure the scale and locate the endpoints for the war, but doesn't count off the spaces to locate an exact midpoint. Notice that student H seems to understand the concept of ± 40, but has further work of finding values within the range of possibility.

Student H

[T]he American Revolution started in 1775 and finished in 1783.
[I]n which year was it halfway through? 1780 X

Show how you know using this number line.

Holly's teacher says the school library has 1000 books.
Holly thinks that the library may have 40 more books.

fw

What is the greatest number of books that the library could have? 1012 X

Show how you figured this out.

Because I don't think the library has 1040 books. It could at least have a little less.

fw fw

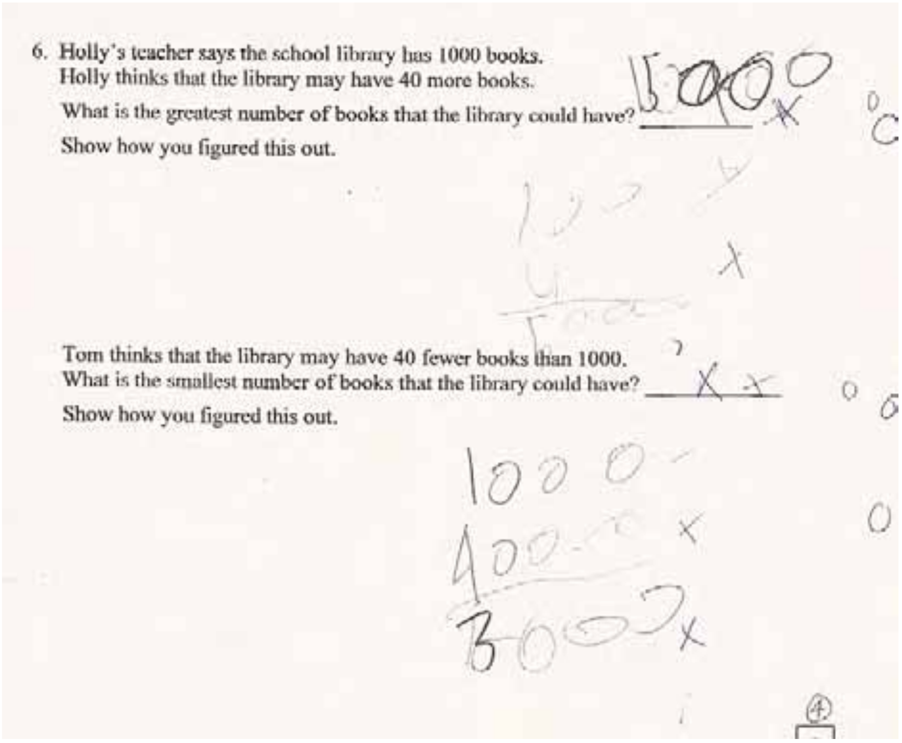
Tom thinks that the library may have 40 fewer books than 1000.
What is the smallest number of books that the library could have? 987 X

Show how you figured this out.

Because I don't think the library could have 960 books, either. It could at least have a little more.

Some students are having difficulty applying ideas about place value to computation. Student I has 4 points on page 1 of the task, but lines up thousands and tens when adding and subtracting. Student J puts the 4 tens in the hundreds column. Student K and L have trouble with place value which leads to procedural errors in using the standard algorithm. Notice Student K also has trouble thinking about what comes after 1778.

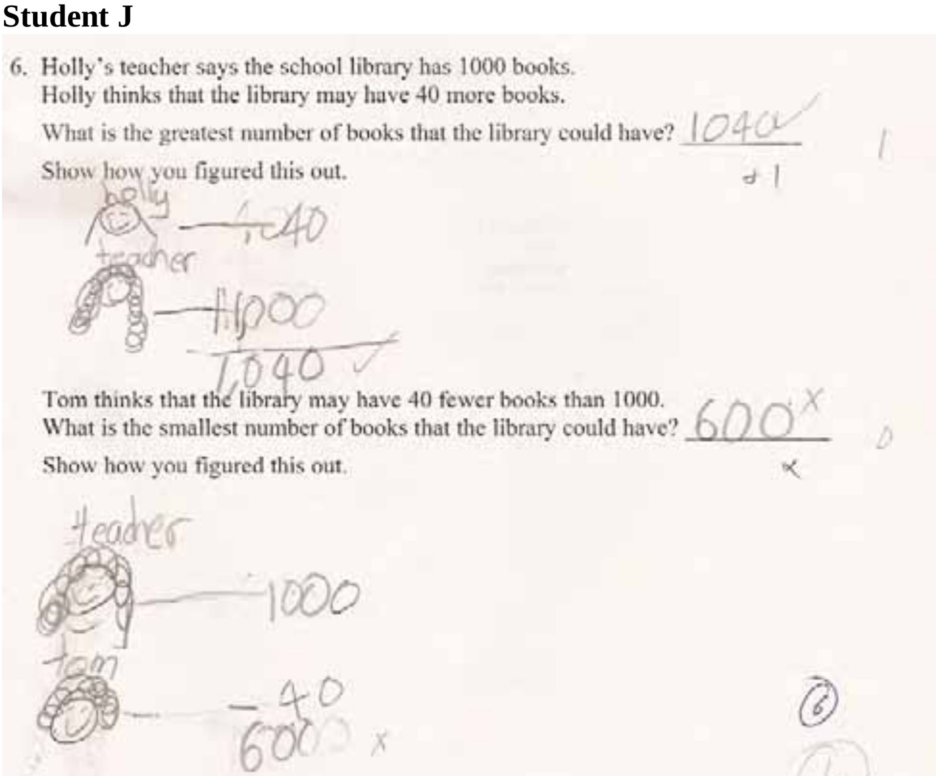
Student I


6. Holly's teacher says the school library has 1000 books.
Holly thinks that the library may have 40 more books.
What is the greatest number of books that the library could have? 5000
Show how you figured this out.

Tom thinks that the library may have 40 fewer books than 1000.
What is the smallest number of books that the library could have?
Show how you figured this out.

1000
400
3000

Student J


6. Holly's teacher says the school library has 1000 books.
Holly thinks that the library may have 40 more books.
What is the greatest number of books that the library could have? 1040
Show how you figured this out.

holly — 40
teacher — +1000
1,040

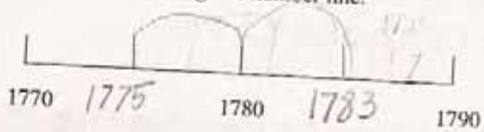
Tom thinks that the library may have 40 fewer books than 1000.
What is the smallest number of books that the library could have? 600
Show how you figured this out.

teacher — 1000
tom — −40
600

Student K

5. The American Revolution started in 1775 and finished in 1783.
In which year was it halfway through? 1778 and a half ✗

Show how you know using this number line. 0



6. Holly's teacher says the school library has 1000 books.
Holly thinks that the library may have 40 more books.
What is the greatest number of books that the library could have? 1,040 ✓ 1
Show how you figured this out.

$$\begin{array}{r} 1,000 \\ +\ \ 40 \\ \hline 1,040 \end{array}$$ ✓

Tom thinks that the library may have 40 fewer books than 1000.
What is the smallest number of books that the library could have? 950 ✗
Show how you figured this out.

$$\begin{array}{r} 1,000 \\ -\ \ 40 \\ \hline 950 \end{array}$$ ✓ 0

Student L

Tom thinks that the library may have 40 fewer books than 1000.
What is the smallest number of books that the library could have? 150 ✗ 0.

Show how you figured this out.

$$\begin{array}{r} 0104 \\ 1000 \\ -\ \ 40 \\ \hline 150 \end{array}$$

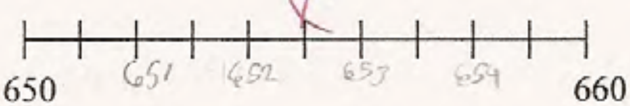
6

6

3rd grade - 2007

Understanding mathematical terminology or academic language is often different from common word usage. Student M relies too heavily on finding and circling key words, rather than teasing out what is being asked in the question. In this particular problem, “and” does not mean the combining of two values, but is identifying two points that are similar because that represent endpoints.

Student M



Put an **X** on the number line, on the place that is halfway between 650 and 660.

What is the number that should be there? 651

Holly knows a pony weighs between 365 pounds and 425 pounds.

Write a possible number for the weight of a pony. 790 pounds

365
+425
790

The activity center swimming pool holds between 1,875 gallons and 1,940 gallons of water.

1875
+1940
3735

Write a possible number for the amount of water the swimming pool holds. 3735 gallons

3950

A school computer could cost between $2,950 and $3,055.

Give three possible prices for the computer.

$ 3,950 $ 4,950 $ 5,950

Student N has trouble distinguishing between the number with the greatest absolute value and adding more books to obtain the greatest possible value. This example shows the subtleties required for reading academic material.

Student N

6. Holly's teacher says the school library has 1000 books.
Holly thinks that the library may have 40 more books.

What is the greatest number of books that the library could have? 1000 X 0

Show how you figured this out.

I know that it is 1000 books because It says what is the grater number and it is 1000.

Tom thinks that the library may have 40 fewer books than 1000.
What is the smallest number of books that the library could have? 40 X 0

Show how you figured this out.

It Says what is the ~~gre~~ smalest number and it would be 40. X

Student Task	Compare and order numbers using values and number lines. Make a reasonable argument to find the midpoint of a range of numbers.
Core Idea 1 Number Properties	**Understand numbers, ways of representing numbers, relationships among numbers and number systems.** • Develop understanding of the relative magnitude of whole numbers and the concepts of sequence, quantity, and the relative positions of numbers.

Mathematics in this task:

- Ability to measure in equal size units on a number line
- Ability to count around tens, hundreds, and thousands
- Ability to find a numerical midpoint and to find and compare numbers within a range
- Understand most and least in the mathematical sense of ± 40.

Based on teacher observations, this is what third graders knew and were able to do:

- Find and name the middle number on a number line (part 1) when the scale marks are drawn in
- Find the most books that could be in a library
- Find points between two numbers

Areas of difficulty for students:

- Subtracting from 1000
- Finding a midpoint between two numbers on a number line, when the intervals are not marked and the endpoints for the task are different from the endpoints on a number line
- Marking off equal intervals, dividing something into ten parts
- Understanding that between does not include the endpoints

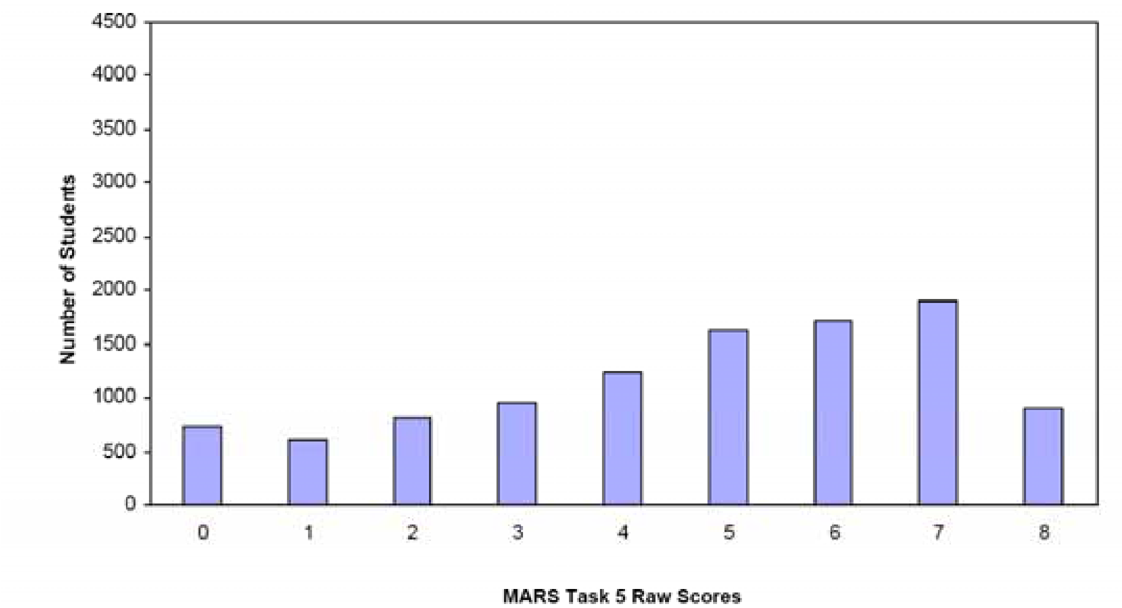
Task 5 - A Question of Numbers

Mean: 4.67 StdDev: 2.32

Table 19: Frequency Distribution of MARS Test Task 5, Grade 3

Task 5 Scores	Student Count	% at or below	% at or above
0	723	6.9%	100.0%
1	603	12.6%	93.1%
2	812	20.4%	87.4%
3	959	29.5%	79.6%
4	1245	41.4%	70.5%
5	1631	57.0%	58.6%
6	1713	73.3%	43.0%
7	1896	91.4%	26.7%
8	904	100.0%	8.6%

Figure 28: Bar Graph of MARS Test Task 5 Raw Scores, Grade 3



The maximum score available for this task is 8 points.
The minimum score needed for a level 3 response, meeting standard, is 4 points.

Most students, about 87%, could find and name the midpoint on a scaled number line. Many students, about 71%, could locate the midpoint on a number line, and find values between two endpoints (parts 2 and 3). More than half, 58%, could also find three values between two endpoints. 26% could also find solutions for thinking about the most and least books in the library. 8% could meet all the demands of the task including scaling their own number line to find a midpoint where the endpoints were not the ends of the number line. 7% of the students scored no points on this task. Only 38% of those students attempted the task.

A Question of Numbers

Points	Understandings	Misunderstandings
0	38% of the students attempted the task.	Time may have been an issue for many students. Some students struggled in trying to find the midpoint between 650 and 660 on the number line. 7.5% of the students marked 656. Some students thought the number was 655.5. Students may have been counting lines instead of spaces to count of the distance.
2	Students could mark and name the midpoint between 650 and 660.	Students had difficulty interpreting between in parts 2 and 3. 7.5% gave endpoints instead. 3% added the endpoints.
4	Students find a midpoint on a number line and give values between two endpoints. (parts 2 and 3)	Students had difficulty naming three values between two endpoints. Almost 18% gave at least one value that was above the highest endpoint. 15% gave values that were too low. 5% included one or both endpoints.
6	Students could find midpoints, locate values in between two endpoints (parts 2,3, and 5), and find the most books that could be located in the library.	About 7% gave the correct answer for most, but showed no calculations or explanation. 7% thought the answer for most was 1000 because it was the greatest number in the problem. Some students had difficulty with place value, getting answers like 5000 or 1400. Students had even more difficulty with the subtraction (1000-40). 4% had the correct answer but no work. 6% thought the answer was 40. 6% had place value errors getting either 60 or 600. Others had place value errors like 3000, 996, or 9504.
7	Students could find midpoints, locate values in between two endpoints, and find the most and the least books that could be located in the library.	Students had difficulty using a number line to find a midpoint when there was no scale. They could figure out how to divide the sections into 10 equal parts and may have only put 2 or intervals. 21% picked 1780, the middle of the given number line but not the middle of the war(1775 to 1783). 7.5% knew the midpoint was 1779, but used the number line incorrectly or did not use the number line. 5% thought the midpoint was 1778. 5% thought the midpoint was 1775 (midpoint for left half of the number line). 4% picked one of the endpoints on the given number line.
8	Students could find midpoints, locate values in between two endpoints, and find the most and the least books that could be located in the library. Students could also mark in intervals on a number line and use it to find the midpoint between two numbers, which were not the endpoints on the number line.	

Implications for Instruction

Students need to understand place value through the thousands and be able to use place value to find numbers falling between two values. Students should be able to use mathematical models, like a number line, to help them think about the size and order of numbers and to help them solve simple problems. A good article that talks about developing number line through measurement is in the NCTM Yearbook, Developing Mathematical Reasoning in Grades K-12: "Mathematical Reasoning within the Context of Measurement"

Students often have trouble counting beyond a certain number:

28,29,30, 40,50, 60 . . .
98,99,100, 105,110,120 . . . .

Students need activities like what number is one more than 399 or what number is one less than 590. Students should also be thinking about what is 10 more or 10 less. This helps them to develop larger ideas about place value and how the number system works.

Students need to understand the logic behind procedures. Put up some examples (without student names) of the place value errors. (See Student I and J) Ask students if these methods are correct. Try to have them develop a convincing argument for their opinions.

Ideas For Action Research:

Try to develop students use of number line to think about quantity of numbers by doing number line talks like the ones on the next page. Also think about using number line to record student strategies for adding and subtracting numbers during number talks. See the examples below for:

Maria has 87 cookies in a box. How many will be left if she eats 18 of them?

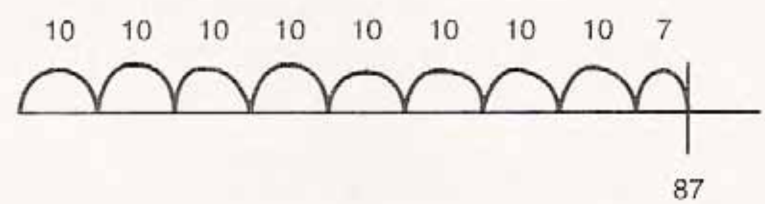


Fig. 8.4. Angie's explanation of 87 cookies

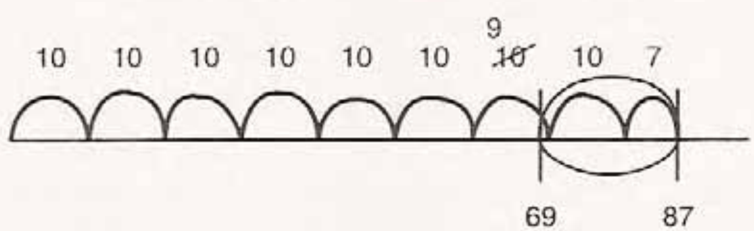


Fig. 8.5. Angie's explanation of 87 cookies and 18 are eaten

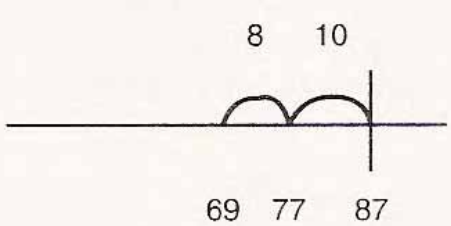


Fig. 8.6. Michael's solution for 87 cookies take away 18 cookies

Number Lines

Purpose:

- To understand relationships between numbers
- To understand the relative magnitude of numbers.

Description:

Students place numbers on a number line. Students use what they know about one number to determine where a second number should be placed. As the types of numbers change and as the scale changes, students must use reasoning skills and their understanding of amounts and quantities to place the numbers.

Materials:

- A large, blank number line easily visible to all students during the routine time
- Attached blackline master of number lines

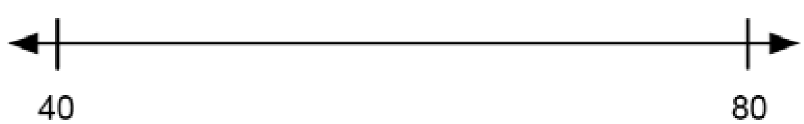
Time: 15 minutes maximum

Caution: Always include arrows on both ends of your number line representations students realize the number line is infinite; we are only looking at a section of the number line.

Directions:

VARIATION 1: ESTIMATION

1. Label 2 marks on the number line (e.g., 40 and 80).



2. Place an arrow somewhere between the 2 marks.

3. The class suggests reasonable values for the number at the arrow.
   The students should give reasons why the numbers they suggest are reasonable (e.g., "It looks like the arrow is about one fourth of the distance between 40 and 80. Since 60 is halfway between 40 and 80 and 50 is halfway between 40 and 60, I think it might be 50.").

Scaffold for Variation 1:

Give the students several numbers to choose from. Students select the number that makes the most sense to them and explain their reasoning. For example:

The arrow is pointing to which of the following numbers? Support your response with a mathematically convincing argument.

85, 49, 78

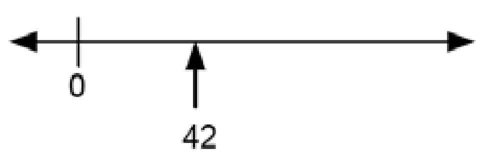
Guiding questions for Variation 1:

- Support your placement with a mathematically convincing argument.
- Name a number that is greater than this number.
- How much greater? Prove it on the number line.
- Name a number that is less than this number.
- How much less? Prove it on the number line.

<u>VARIATION 2: ESTIMATION</u>

1. Label the mark on the left with a zero.

2. Tell the students the arrow is pointing to a particular number (e.g., The arrow is pointing to 42).



3. Ask where other numbers would be. This helps students look at the relative positions of values. For example:
   About where would 83 be?
   About where would 21 be?
   About where would 31 be?
   Justify your answers with mathematically convincing arguments.

Activities from website for San Diego School District: Mathematics Dept. Third grade

Reflecting on the Results for Third Grade as a Whole

Think about student - Work through the collection of tasks and the implications for instruction. What are some of the big misconceptions or difficulties that really hit home for you?

If you were to describe one or two big ideas to take away and use for the planning for next year what would they be?

What were some of the qualities that you saw in good work or strategies used by good students that you would like to help other students develop?

Three areas that stood out for the Collaborative as a whole for third grade mathematics were:

1. Understanding Scale – Students had difficulty working with a "unit" or equal size group. In Square Patterns we wanted students to notice that there were four arms so white squares grew in groups of 4. In Parking Cars, students could often read a value off a graph with a scale or intervals of 10, but couldn't use scale to do comparison subtraction or add data to the graph. They also had difficulty with thinking about what the value of a bar between the grid lines represented. The same is true in A Question of Numbers. Students had difficulty with understanding between. It is not clear if this is a counting issue, place value issue, or vocabulary issue. Teachers should be encouraged to investigate these ideas in their classrooms. Students had difficulty subdividing the number line into equal size groups or units.
2. Place Value – Place value issues arose in student work for Adding Numbers. Students did not know where to put numbers when writing their own addition and subtraction problems. The process of writing your own problem to solve is different than working a problem that is already set up on a worksheet. Similar problems arose in a Question of Numbers when students had to write their own problems for adding and subtracting 40 from 1000. Students also had difficulty finding values that are between. Some students only considered the leading digit when comparing values.
3. Attributes - In Square Patterns students were to look at attributes of a growing pattern and try to describe them. Students who could do this well were able to make generalizations that helped them solve other parts of the task. In What Shape? students struggled with finding attributes of shape: They may have not used all the attributes to identify a shape. They may have trouble giving enough attributes to define a square. Students had difficulty describing similarities and differences in the final two shapes.

Performance Assessment Task
A Question of Numbers Grade 3
This task challenges a student to use an understanding of numbers and place values through thousands place, ways of representing numbers, and relationships among numbers to order and compare numbers. A student must demonstrate understanding of scale and equal value to situate numbers on a number line. A student must make sense of the relative size of numbers and locate numbers that fall between, above or below given values. Students must demonstrate understanding of operations with numbers within thousands.
Common Core State Standards Math - Content Standards
<u>Number and Operations in Base Ten</u> **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 3.NBT.1 Use place value understanding to round whole numbers to nearest 10 or 100. 3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
Common Core State Standards Math – Standards of Mathematical Practice
MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving **MP.6 Attend to precision.** Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.
Assessment Results
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
3	2007	8	4	70%