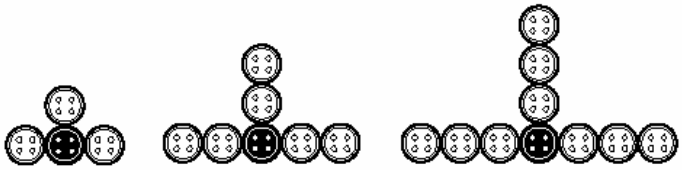
Buttons

This problem gives you the chance to:
• describe, extend, and make generalizations about a numeric pattern

Gita plays with her grandmother's collection of black and white buttons.
She arranges them in patterns.
Her first 3 patterns are shown below.



Pattern 1 Pattern 2 Pattern 3 Pattern 4

1. Draw Pattern 4 next to Pattern 3.

2. How many **white** buttons does Gita need for Pattern 5 and Pattern 6?

Pattern 5 _____ Pattern 6 _____

Explain how you figured this out.

_____

_____

_____

3. How many buttons in all does Gita need to make Pattern 11?

_____

Explain how you figured this out.

_____

_____

_____

_____

4. Gita thinks she needs 69 buttons in all to make Pattern 24.

How do you know that she is **not** correct?

_____

_____

_____

How many buttons does she need to make Pattern 24? _____

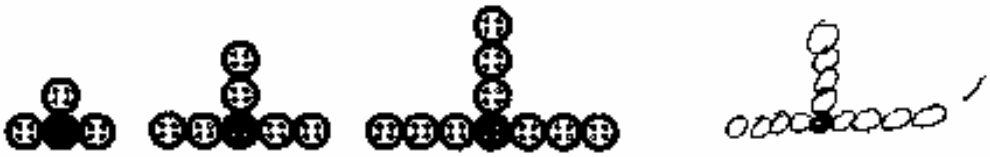
8

Buttons — Test 5 Form A Rubric

The core elements of performance required by this task are: • describe, extend, and make generalizations about a numeric pattern Based on these, credit for specific aspects of performance should be assigned as follows:	Points	Section Points
1. Draws correct Pattern 4.	1	1
2. Gives correct answers as: Pattern 5 = **15** white buttons Pattern 6 = **18** white buttons Gives explanation such as: Pattern 5 has 3 rows of 5 white buttons = 15 white buttons and Pattern 6 has 3 rows of 6 white buttons = 18 white buttons. *Special case:* Gives answers 16 and 19 with correct explanations including black buttons.	 1 1 1 2 s c	3
3. Gives correct answer as: **34** Gives explanation such as: Pattern 11 has 3 rows of 11 white buttons = 33 white buttons and 1 black button. *Special case:* Gives answer as 33 with a correct explanation for the white buttons.	 1 1 1 s c	2
4. Gives explanation such as: Pattern 24 needs 3 rows of 24 white buttons = 72 white buttons and 1 black button, 73 buttons in all. *Accept alternative correct explanations such as: 69 is divisible by 3, so it cannot be correct.* Gives correct answer as: **73**	 1 1	2
Total Points		**8**

Looking at Student Work – Buttons

Many students at this grade level are able to see the pattern and form a generalization in words or in number algorithms. These generalizations could easily be converted to algebraic symbols at later grade levels. Student A has a nice description of how the pattern grows and an algorithm for find the total number of buttons in part 4.



Pattern 1 Pattern 2 Pattern 3 Pattern 4

1. Draw Pattern 4 next to Pattern 3.

2. How many white buttons does Gita need for Pattern 5 and Pattern 6?

Pattern 5 15 buttons Pattern 6 18 buttons

Explain how you figured this out.

For each pattern there is the same number of white buttons on 3 sides of the black button, so you count the number of buttons on one side of the black buttons and multiply that by three.

3. How many buttons in all does Gita need to make Pattern 11?

34

Explain how you figured this out.

I did the same process in the problem above but I added 1 to my answer for the black button.

Student A, part 2

4. Gita thinks she needs 69 buttons in all to make Pattern 24.

How do you know that she is not correct?

Because if you follow the pattern of multiplying 3 to the number of buttons on one side and plus one for black button. it doesn't equal 69.

How many buttons does she need to make Pattern 24? 73 buttons

$$\begin{array}{r} 24 \\ \times\ 3 \\ \hline 72 \\ +\ 1 \\ \hline 73 \end{array}$$

For further examples of making generalizations and algorithms, look at the work of Student B and C.

Student B

4. Gita thinks she needs 69 buttons in all to make Pattern 24.

How do you know that she is not correct?

I know because (24 x 3) + 1 does NOT equal 69.

How many buttons does she need to make Pattern 24? 73

$$\begin{array}{r} 24 \\ \times\ 3 \\ \hline 72 + 1 = 73 \end{array}$$

Student C

Explain how you figured this out.

There are 11 white buttons on each line. So 11×3= 33. Then plus ✓
one black bead which is 34.

33+1= 34 buttons.

How do you know that she is not correct?

24
×3
72
+1 black button
73

While Student D does a thorough explanation of the growing pattern and finding her answers in part 2 and 3, the explanation about multiplying is unclear in part 4. However the work at the top of page shows a understanding of the process for finding the number of buttons in pattern 24.

Student D

Explain how you figured this out.

I figured this out because on pattern four we ✓
put 4 white buttons in each side and 4 buttons in the ✓
top. So I need 5 for each side in paterns and 5 on top
and I need 6 for each side in pattern 6 and plus 6 on top. ✓1

3. How many buttons in all does Gita need to make Pattern 11?

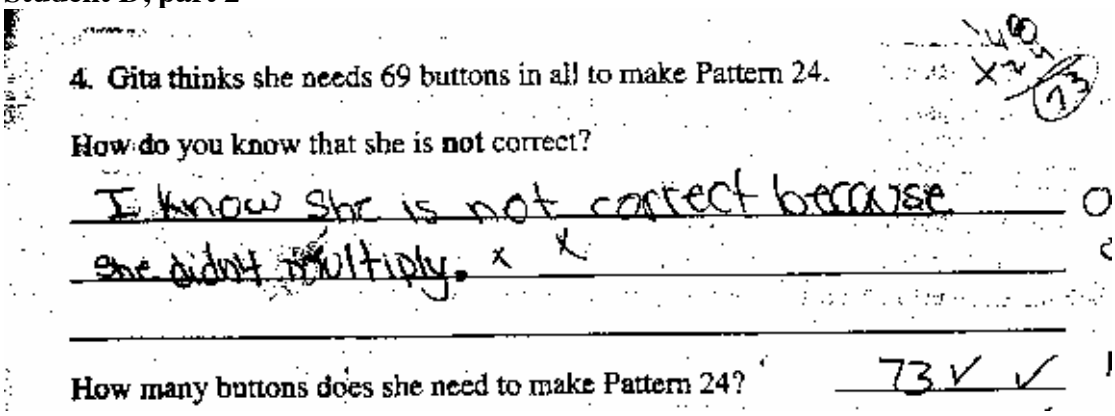
34 ✓✓ 1

✓1

Explain how you figured this out.

I figured this because 3 x 11 = 33 plus ✓
1 black button = 34. ✓

Student D, part 2


4. Gita thinks she needs 69 buttons in all to make Pattern 24.

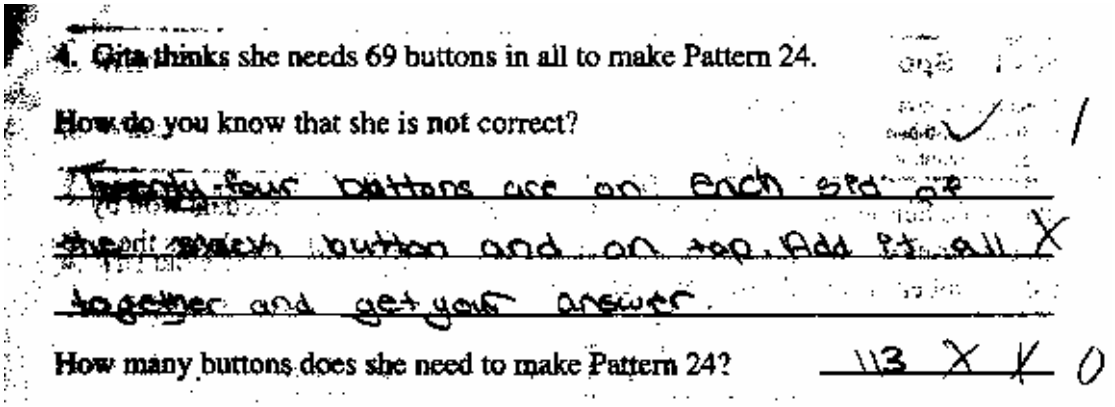
How do you know that she is **not** correct?

I know she is not correct because she didn't multiply.

How many buttons does she need to make Pattern 24? 73

Student E has a very good generalization of the process for finding the total buttons in pattern 24. Unfortunately the student does not seem to use it to get the final answer.

Student E


4. Gita thinks she needs 69 buttons in all to make Pattern 24.

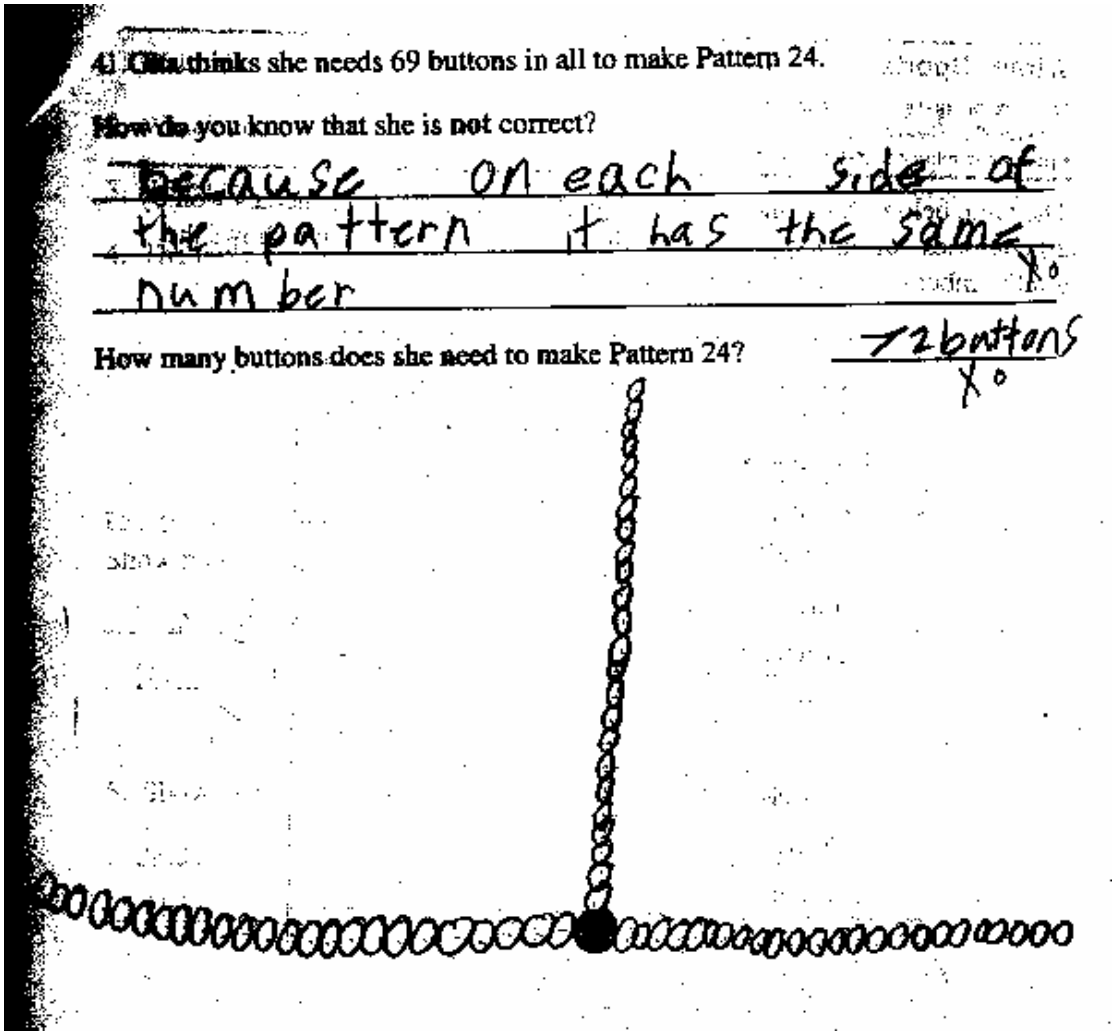
How do you know that she is **not** correct?

[illegible] buttons are on each side of the [illegible] and on top. Add it all together and get your answer.

How many buttons does she need to make Pattern 24? 113

Some students can find the answer or come close to a solution, but use strategies that are inefficient, tedious, and may lead to errors. Student F uses a drawing and counting strategy. Student G does not see the complete pattern and solves for the white buttons every time. Student G uses repeated addition, which is a correct process, but makes a calculation error. Student H uses graphing, but because of the inaccuracies of the graph, the student's answer is incorrect.

Student F

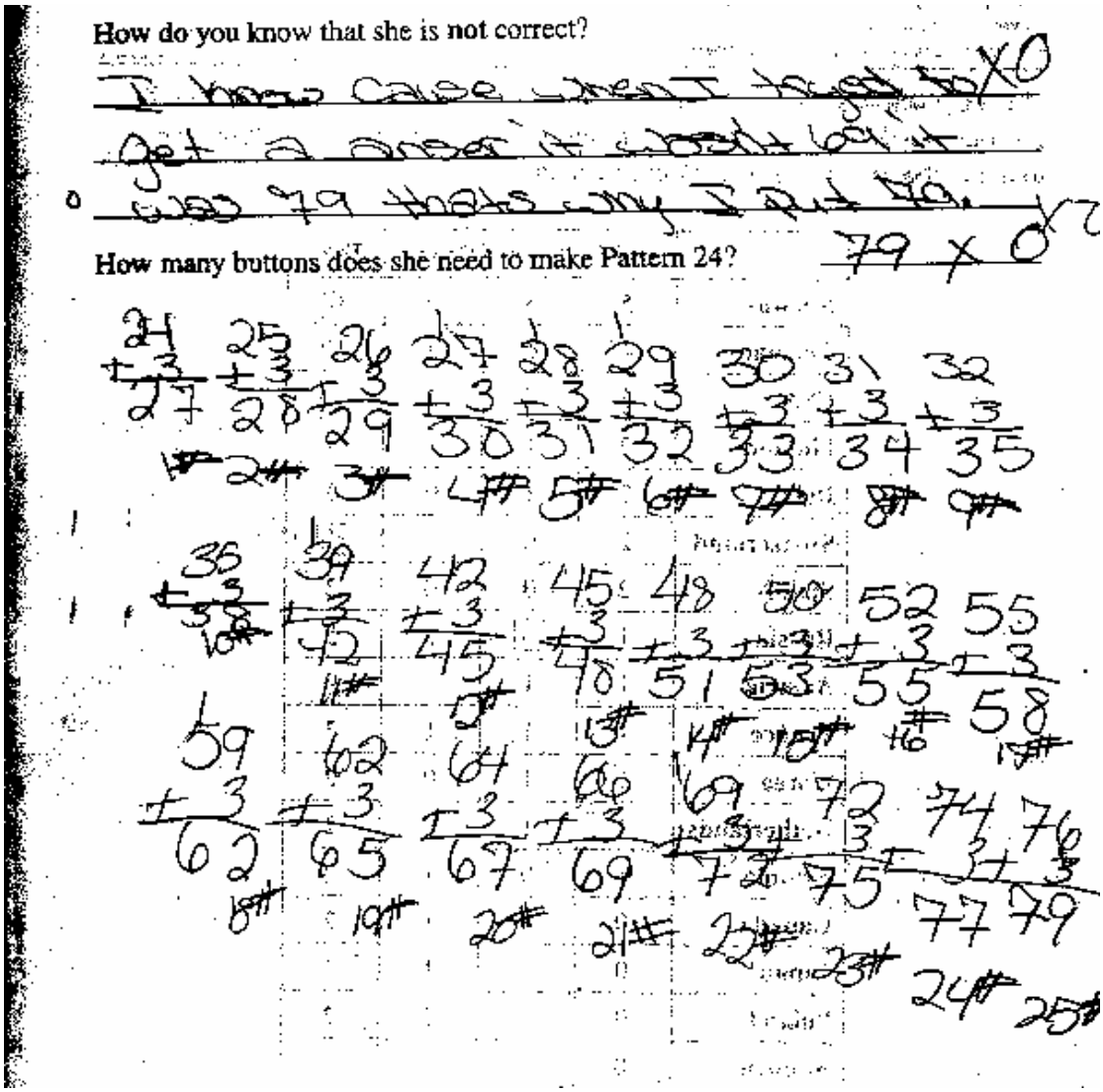

4) [redacted] thinks she needs 69 buttons in all to make Pattern 24.

How do you know that she is not correct?

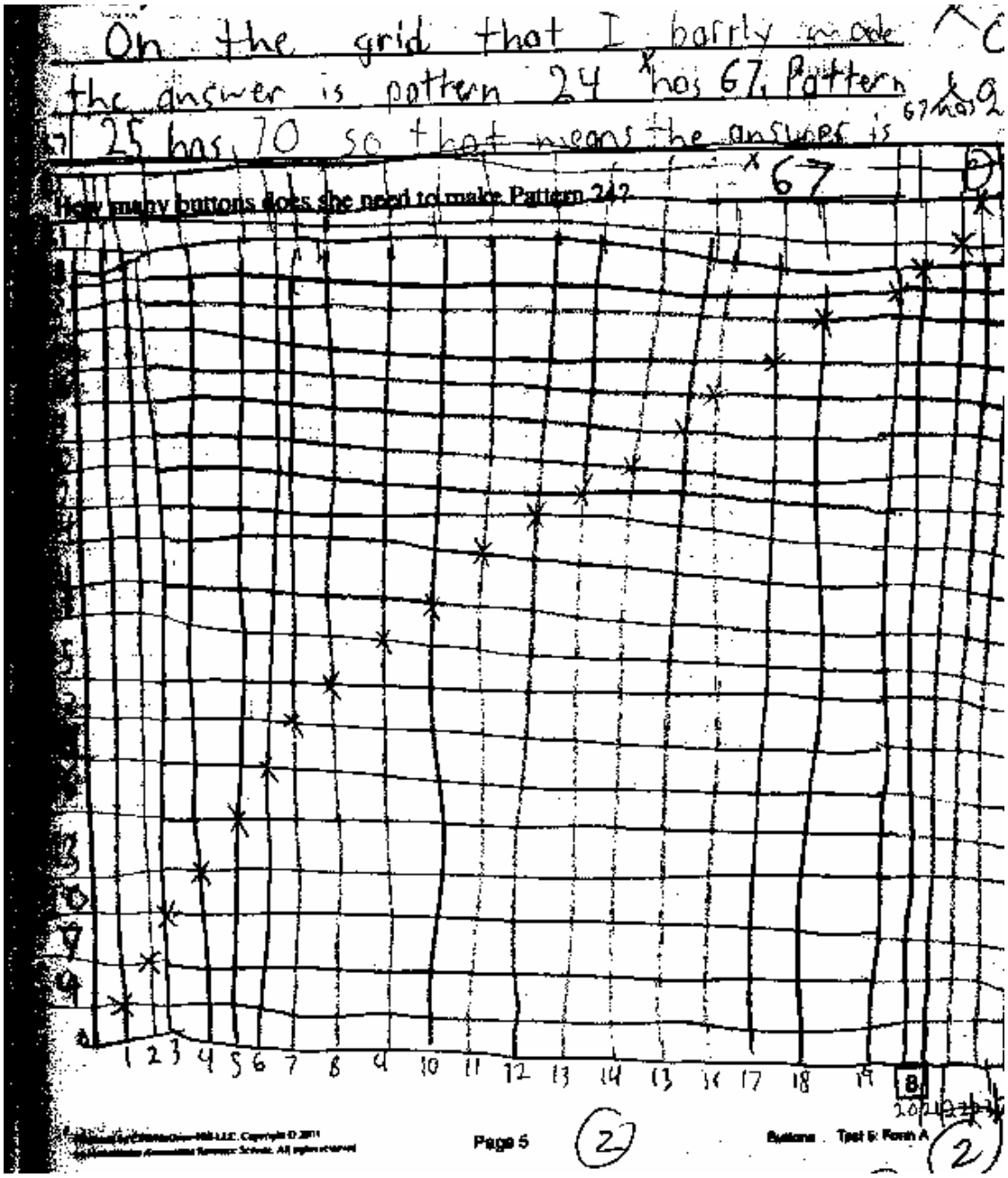
because on each side of the pattern it has the same number

How many buttons does she need to make Pattern 24? 72 buttons

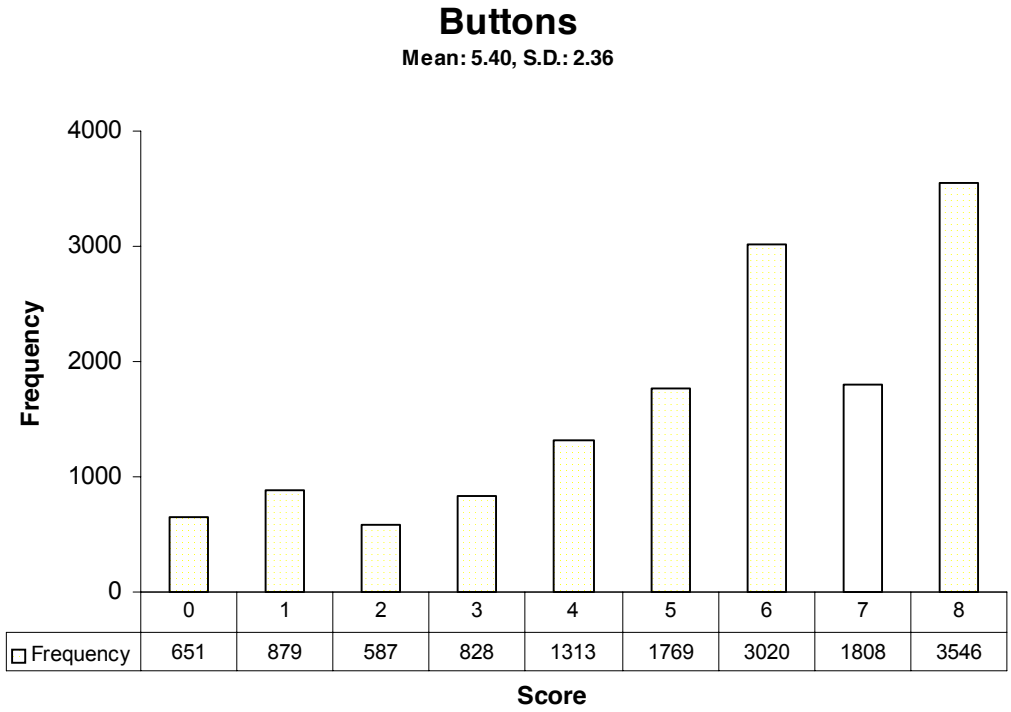
Student G



Student H



Teacher Notes:



Score:	0	1	2	3	4	5	6	7	8
% < =	4.5%	10.6%	14.7%	20.4%	29.6%	41.9%	62.8%	75.4%	100.0%
% > =	100.0%	95.5%	89.4%	85.3%	79.6%	70.4%	58.1%	37.2%	37.2%

The maximum score available for this task is 8 points.
The cut score for a level 3 response is 4 points.

Most students (about 89%) could draw pattern 4 and give the correct number of buttons for pattern 5 and pattern 6. Many students (about 80%) could draw and extend the pattern and explain in words how the patterned worked. More than half the students (about 60%) could draw the pattern, extend the pattern for 5 and 6 and explain how it grew, find and explain the number of white buttons for pattern 11, and could find the total buttons for pattern number 24. Almost 26% of the students met all the demands of the task. About 5% of the students scored no points on this task.

Buttons

Points	Understandings	Misunderstandings
0	Most students attempted this task.	
3	Students could make a drawing of the next pattern in a sequence. They could also find the number of white buttons for pattern 5 and 6.	Students had difficulty explaining how they figured out the white buttons in the pattern. They said things like, "I added", without specifying what or why.
6	Students could find and extend a pattern. To find a solution in part 4,	They did not convert from white buttons to finding the pattern for all buttons in part 3 and 4. 37% of all the students multiplied 24 by 3 in part 4, but did not add on the black button.
7	31% of all students used the expression of 24 times 3 plus 1 to solve for part 4. Another 7% used 24+24+24+1.	Some students used inefficient strategies, which may have led to errors. 13% drew a picture. 8% made a table and extended it, 3% just kept adding 3, and some students drew a graph.
8	Students could continue a pattern using a drawing. They could also find rules for extending it without drawing.	

Based on teacher observations, this is what fifth grade students know and are able to do:

- Continue a pattern using pictures and numbers
- Explain how a pattern grows and use that algorithm to solve for larger numbers in the pattern

Areas of difficulty for fifth graders, fifth grade students struggled with:

- Distinguishing between part of a pattern and the whole pattern
- Explaining a pattern in words

Teacher notes:

Questions for Reflection on Buttons:

- What are some rich problems that your students have done this year? What are some good resources for pattern problems?
- Do you ask questions like: "What stays the same?" and "What changes?" to help students develop the ability to form generalizations?
- Do students have opportunities to connect their number sentences to geometric patterns and share how they visualize the growth pattern?

Look carefully at your student work. What strategies did they use in Part 4?

Draw a picture	24 x 3 or 24+24+24	24 x 3 + 1 or 24+24+24+1	Extend a table	Repeated addition of 3	Used a doubling strategy from a previous part of the problem

Teacher Notes:

Instructional Implications:

Fifth grade students need more experiences that require them to move beyond drawing the next figure in the pattern so that they analyze the pattern and represent the growth numerically. Fifth graders need to move beyond thinking about "what comes next?" to thinking about the problem as a whole: this involves generalizing what is happening with the growth, but need not necessarily involve variables or algebraic equations. Being able to see what remains the same and what changes in a pattern helps students develop algebraic thinking and the ability to make a generalization. Asking questions about how the pattern changes helps students to move beyond counting and drawing strategies to rules that will solve for any number in the pattern.

Performance Assessment Task

Buttons

Grade 5 task aligns in part to CCSSM grade 4

This task challenges a student to describe, extend, and make generalizations about a geometric pattern. Students must be able to extend the pattern in drawings and making nonconsecutive jumps. Students must be able to develop a mathematical argument for why a number does not fit the pattern.

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking

Generate and analyze patterns.

4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.

Common Core State Standards Math – Standards of Mathematical Practice

MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even through they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MP.7 Look for and make use of structure.

Mathematically proficient students try to look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
Grade 5	2003	8	4	80%