Coffee
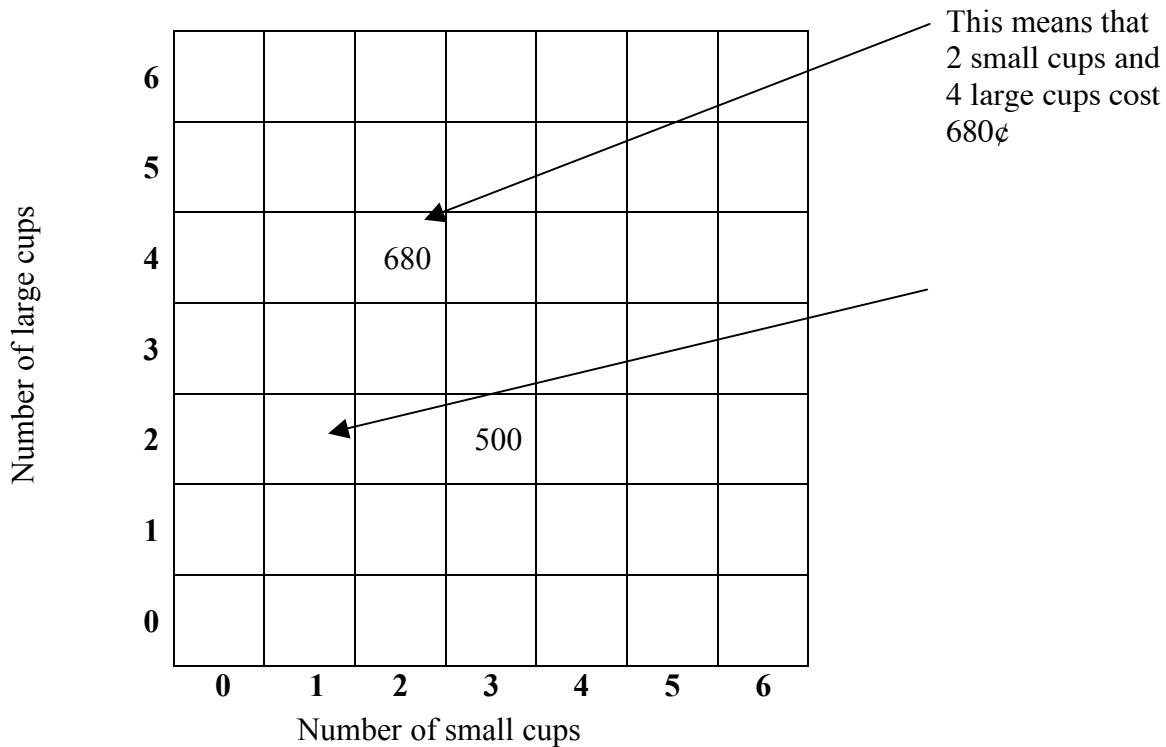
This problem gives you the chance to:
• use a chart to solve simultaneous equations

This chart shows the cost, in cents. of different numbers of small and large cups of coffee.

This means that
2 small cups and
4 large cups cost
680¢

Number of large cups

680

500

Number of small cups

1. Explain what the number 500 in the chart means.

_____

2. Use the information in the chart to find the cost of a small cup of coffee and the cost of a large cup of coffee. Show how you figured it out.

Small cup of coffee costs_____cents   Large cup of coffee costs _____cents

3. What number should go in the empty box that the arrow is pointing to. _____
   Explain your work.

   _____

   _____

   _____

Coffee		Rubric
The core elements of performance required by this task are: • use a chart to solve simultaneous equations Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer such as: the cost of 3 small cups of coffee and 2 large cups of coffee is 500¢.	1	1
2. Gives correct answer: small **80¢,** large **130¢** Shows work such as: solving simultaneous equations *Partial credit* Some correct work	2 x 1 2 (1)	 4
3. Gives correct answer: **340** and Gives correct explanation such as: The cost of one small cup and two large cups. (is half the cost of two small cups and four large cups**.)**	2ft	 2
Total Points		**7**

Coffee

Work the task and look at the rubric. What are the big mathematical ideas being assessed in the task?

_____

Look at student work on part 1, explaining the 500. How many of your students put:
- Complete explanation, 2 large and 3 small cups cost 500 cents?_____
- Cost of the coffees?(Ignored the units cents or didn't quantify the cups of coffee)? _____
- Misread the scale:
  - 2 large and 2 small?_____
  - 3 large and 2 1/2 small?_____
  - 2 large and 1 small?_____
  - Other?_____

Now look at student work on part 2, finding the cost of small and large coffees. How many of your students:
- Could solve simultaneous equations to find the cost?_____
- Tried to set up an equality between the 2 equations?_____
- Used guess and check or substitution?_____
- Gave answers with no work?_____
- Did not attempt this part of the task?_____

How often do students get opportunities to apply their algebraic skills in context?
Why do you think guess and check is such a prevalent strategy?
How can you design a lesson that would help students see the reason to leave behind comfortable arithmetic and make it worthwhile to learn new mathematics?

Now look at work in part 3, finding the cost of the empty square. How many students:
- Could use the correct or incorrect solutions in part 2 were able to read the scale and calculate the cost of 2 large and 1 small coffee?_____
- Used proportional reasoning to find the cost?_____
- Tried to estimate using one of the values in the table?_____
- Only looked at the horizontal or vertical scale, but not both, to compute a value?

Why do you think students struggled with part 3? What was confusing for them? What misconceptions did they have?

Looking at Student Work on Coffee

Student A uses equations to find the cost of small and large coffee. The student is competent computing with negative numbers. In part 3 the student writes an equation and uses substitution to find the cost represented by the empty box.

Student A

This means that 2 small cups and 4 large cups cost ~~680¢~~

680

500

$$2s + 4L = 680$$
$$-2(3s + 2L = 500)$$
$$-6s + 4L = 1000$$
$$-4s = 320.00$$
$$s = 8 \quad b.od$$

1. Explain what the number 500 in the chart means.

   3 small cup and 2 large cups cost 500 ¢ ✓ |

2. Use the information in the chart to find the cost of a small cup of coffee and the cost of a large cup of coffee. Show how you figured it out.

   Small cup of coffee costs ⟨80⟩ ✓ cents   Large cup of coffee costs 130 ✓ cents

   $$3(2s + 4L = 680)$$
   $$-2(3s + 2L = 500)$$
   $$12L = 2040$$
   $$-4L = -1000$$
   $$8L = 1040$$

Coffee

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Student A, part 2

3. What number should go in the empty box that the arrow is pointing to. __340 ¢__ ✓
   Explain your work.

the small cup cost 80 ¢ and the Large
cup cost 130 ¢ and there are 1 small cup
and 2 large cups so the total cost is 340 ¢

$1s + 2L = y$

$S = 80$
$L = 130$

$\begin{array}{r} 260 \\ + 80 \\ \hline 340 \end{array}$ ✓

Student B understands the table, shown by the clear labeling next to the arrow. The student does not know how to set up equations in part 2. Student B uses proportional reasoning to find the cost represented by the empty box.

Student B

The cost of 2 large cups and 1 small cup of coffee

1. Explain what the number 500 in the chart means.

It means thats how many cents it cost for 3 small cups and 2 large cups of coffee

2. Use the information in the chart to find the cost of a small cup of coffee and the cost of a large cup of coffee. Show how you figured it out.
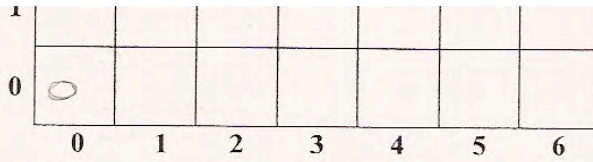
Small cup of coffee costs_____cents   Large cup of coffee costs _____cents

I don't understand this problem

Student B, part 2

3. What number should go in the empty box that the arrow is pointing to. ___340¢___ ✓
   Explain your work.

because there are half as many large cups and small cups in the blank box as there cups for 680¢

Student C tries to use the 2 equations to set up a new equality. The problem with this strategy is that there are always 2 variables. The student at some point needs to use guess and check to get the values. The student can use the values found from guess and check to find the cost of the empty box.

Student C

0 | ○

| 0 | 1 | 2 | 3 | 4 | 5 | 6 |

Number of small cups

1. Explain what the number 500 in the chart means.

3 small cups and 2 large cups = 500 ¢ ✓

2. Use the information in the chart to find the cost of a small cup of coffee and the cost of a large cup of coffee. Show how you figured it out.

80

94   130
      109
80   121
84   124
133   $3x + 2y = 500$

$2x + 4y = 680$

Small cup of coffee costs __80__ ✓ cents   Large cup of coffee costs __130__ ✓ cents

$3x + 2y - 500 = 2x + 4y - 680$
          $+680$ $+680$

$3x + 2y - 180 = 2x + 4y$
$-2x$ $-2x$

$x + 2y - 180 = \dfrac{4y}{4}$

(1) 0

$\frac{1}{4}x + \frac{1}{2}y - 45 = y$
$-\frac{1}{2}y$

$2 \cdot \frac{1}{4}x - 45 = \frac{1}{2}y$

$\frac{1}{2}x - 90 = y$ $\frac{1}{2}$

Page 8   Coffee

Student D tries to use a proportion to solve for the cost of large and small coffees in part 2. Like Student C the student has formed an equation that still has two variables and therefore can't be solved.

Student D

Number of small cups

1. Explain what the number 500 in the chart means.

   3 small cups and 2 large cups costs 500¢   ✓   1

2. Use the information in the chart to find the cost of a small cup of coffee and the cost of a large cup of coffee. Show how you figured it out.

   Small cup of coffee costs ___✗___ cents   Large cup of coffee costs ___✗___ cents   0

   y = cost of large cup
   x = cost of small cup

   $$\frac{2x+4y}{680} \; \cancel{=} \; \frac{3x+2y}{550}$$

   $$1000x + 2000y = 2040x + 1360y$$

   0

Most students do not understand how to use algebra to find the cost of the coffees. Instead Student E makes an organized list and uses guess and check to help find the cost. The student is then able to use the costs to find the missing value in the table. The student relies on arithmetic skills to by-pass the algebra.

Student E

This chart shows the cost, in cents, of different numbers of small and large cups of coffee.

60 40
+4 ×2
240 80

300

This means that
2 small cups and
4 large cups cost
680¢

80 60 80
×4   2 ×2
320 120 160

420

140 80
×4   ×2
560 160
+160
720
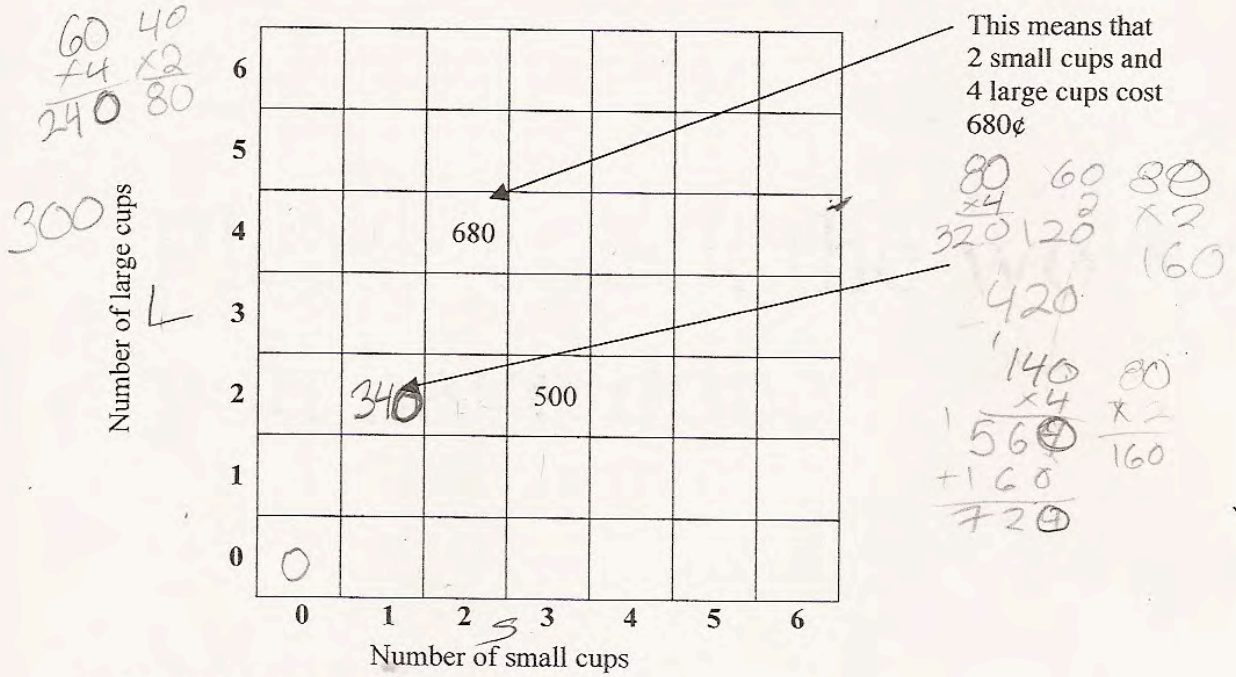
1. Explain what the number 500 in the chart means.

The cost of 3 small cup & 2 large cups ✓

2. Use the information in the chart to find the cost of a small cup of coffee and the cost of a large cup of coffee. Show how you figured it out.

Small cup of coffee costs ___80___ cents   Large cup of coffee costs ___130___ cents

L | S
60 40
100 80
80 60
120 80
140 80
130 90
130 80 ✓

130   90
×4   ×2
520   180

130   80
×2
260

80
×3
240

130   80
×4   +2
520   160
160

560
240
500

Page 8   Coffee

Student F is able to set up ratios to find the missing value in the table. The thinking for this is shown in the section 2. The student attempts to use algebra in part 2 to find the cost of the coffees. However the student doesn't see a second equation. Instead the student tries to add cups of coffee to make a second equation, $x + y = 6$.

Student F

340/2)680
680

2 small cups and 4 large cups cost 680¢

$\frac{2}{4} = 680¢$

680

340   500
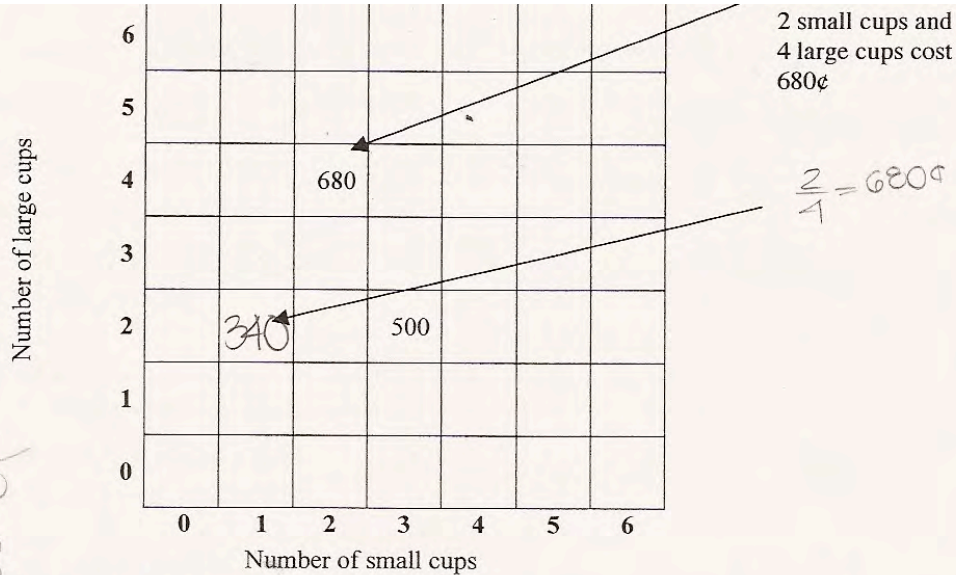
1. Explain what the number 500 in the chart means.

It means 3 small cups and 2 large cups cost 500¢ ✓

2. Use the information in the chart to find the cost of a small cup of coffee and the cost of a large cup of coffee. Show how you figured it out.

Small cup of coffee costs ___328___ cents   Large cup of coffee costs ___334___ cents

X = small cup
Y = large cup

$\frac{2}{4} = 680¢$ $\frac{3}{2} = 500¢$

$\frac{1}{2} = 340¢$

$x + y = 6$ $x = 6 - y$

$2x + 4y = 680¢$

$2(6-y) + 4y = 680$

$12 - 2y + 4y = 680$

$\frac{12 + 2y}{12} = \frac{680}{12}$

$2x + 4(334) = 680$

$2x + 1336 = 680$

$2x = -656$ X

$2y = 668$ Y = 334

Page 8

Coffee

Student G makes the same assumption that the coffees can be added together to make the second equation. When Student G works out the cost of the coffee in the empty box, the size of the answer is way out of scale with other numbers in the table. *Why doesn't the student use this as a trigger that something is wrong? Why do you think students are doing this? What is their misunderstanding? What prompt could you pose to these students to get them to rethink their strategy?*

Student G
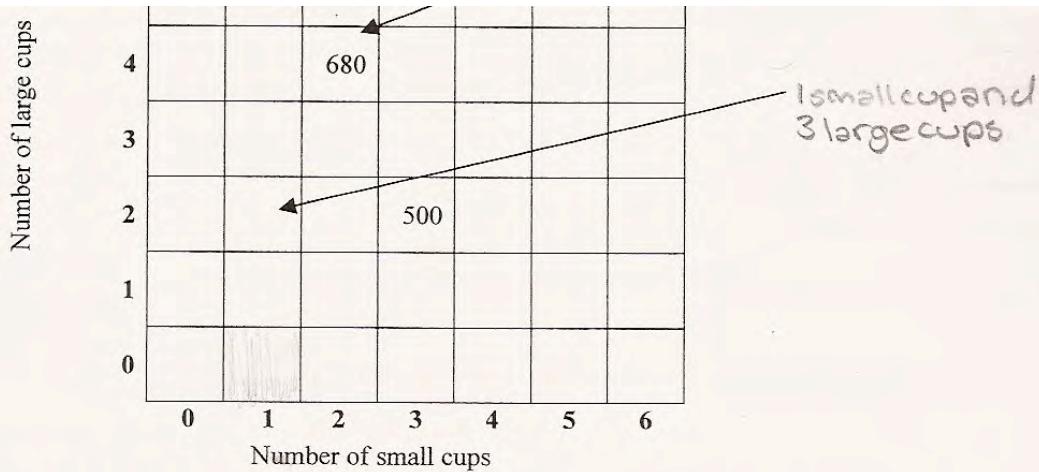
1. Explain what the number 500 in the chart means.

   3 Small cups and 2 large cups cost 500¢.

2. Use the information in the chart to find the cost of a small cup of coffee and the cost of a large cup of coffee. Show how you figured it out.

   Small cup of coffee costs __328__ cents   Large cup of coffee costs __334__ cents

   x = small cup   y = large cup

   2x + 4y = 680   x + y = 6

   $\frac{2x}{2} = \frac{-4y + 680}{2}$ -2y + 340 + y = 6

   $\boxed{x = -2y + 340}$ -y = -334

   y = 334

   x = -2(334) + 340

   x = -668 + 340

   x = -328

   A small cup costs 328 cents and a large cup costs 334 cents.

Page 8   Coffee

Student G, part 2

3. What number should go in the empty box that the arrow is pointing to. ___1330¢___ X   0

   Explain your work.

   First, I took the amount a large and small cup cost which was 334¢ and 328¢. I multiplyed 334 by 3 and got 1002. Then I added 328 (the small cup's amount) and 1002 together. I finally had 1330 as the solution.

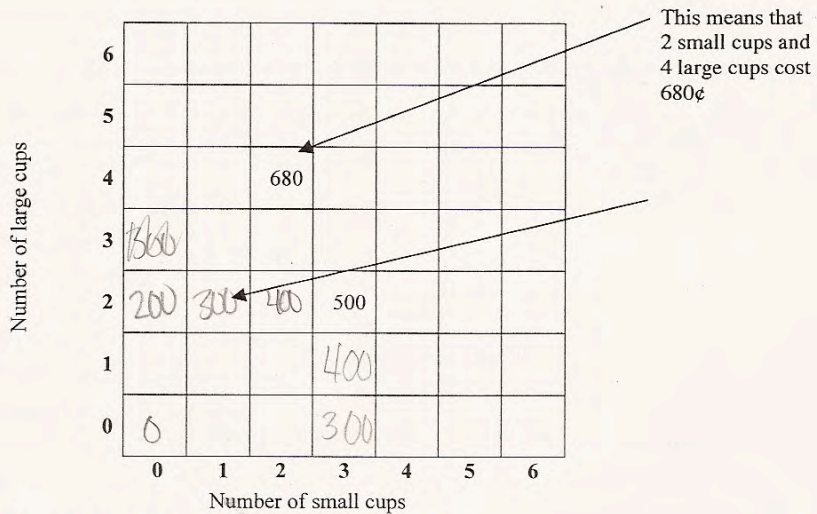 $\begin{array}{r} 3\overset{1}{2}8 \\ +1002 \\ \hline 1330 \end{array}$ $\begin{array}{r} 334 \\ \times\ 3 \\ \hline 1002 \end{array}$ X

Student H tries to make guesses on scale. The student uses 100 as a unit. *What evidence would help the student see the problem with this scale?*

Student H

This means that 2 small cups and 4 large cups cost 680¢

1. Explain what the number 500 in the chart means.

That 3 small cups + 2 large cups costs 500¢. ✓ 1

2. Use the information in the chart to find the cost of a small cup of coffee and the cost of a large cup of coffee. Show how you figured it out.

Small cup of coffee costs _____ ✗ cents   Large cup of coffee costs _____ ✗ cents

Page 8

Coffee

Student H, part 2

3. What number should go in the empty box that the arrow is pointing to. __300__ X
   Explain your work.
   __Adding and subtracting 100's to the graph__ X

Student I also tries to find a scale. The student looks at one scale at a time to find equal units to get to the 500. Unfortunately this method ignores the idea that both scales contribute to the 500. In part 3 the student tries to use the same logic to find the value of the empty box, but finally adds the two prices together. This ignores the quantity of large and small coffees.

Student I

This chart shows the cost, in cents, of different numbers of small and large cups of coffee.
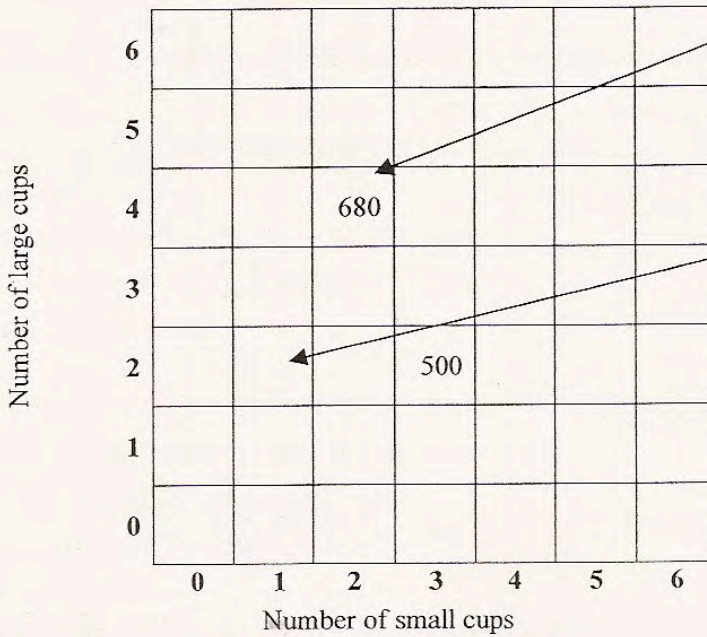
This means that 2 small cups and 4 large cups cost 680¢

6.80

Number of large cups

680

500

Number of small cups

1. Explain what the number 500 in the chart means.

This means that 3 small cups and 2 large cups cost 500¢

2. Use the information in the chart to find the cost of a small cup of coffee and the cost of a large cup of coffee. Show how you figured it out.

Small cup of coffee costs ___125___ cents   Large cup of coffee costs ___250___ cents

Coffee

Page 8

Student I, part 2

3. What number should go in the empty box that the arrow is pointing to. __375__ ✗

   Explain your work.

   Since another box had the number 500 and also had 2 large cups of coffee, I divided 500 by 2 and got 250¢ for the cost of the large cups. I then divided 250¢ by 2 and got 125¢ and I added 125¢ and 25¢ and got 375¢

73.3
6)500
48/20/16

250 350 450 500

166.6
3)500
.3↓
20

250
2)500

14
250
-166
084

250
+84
334

125
2)250
2↓
05

$$\frac{2Y}{2} = \frac{500}{2}$$

$$Y = 250$$

6
12
18
24
32
40
48

250
2)500

250
2)500
-4
10

(250)

$$\frac{4X+2Y}{4} = \frac{680}{4}$$

$$2Y = 170$$

170
× 2
340
Y=340

250
+125
375

170
4)680
-4
28

$$4X+2Y = \frac{680}{2} \frac{}{2}$$

$$4X = 340$$

340
4
-160
340

7 ①

Page 9

Coffee

STOP HERE!

Student J scores no points on the task. In part 1 the student confuses the scales. The student does not connect the numbers in the box with money in part 2. The student tries to estimate the value of the empty box in part 3 by comparing it with the 680, but ignores the 500. *Why should the student know that 580 is too large?*

Student J
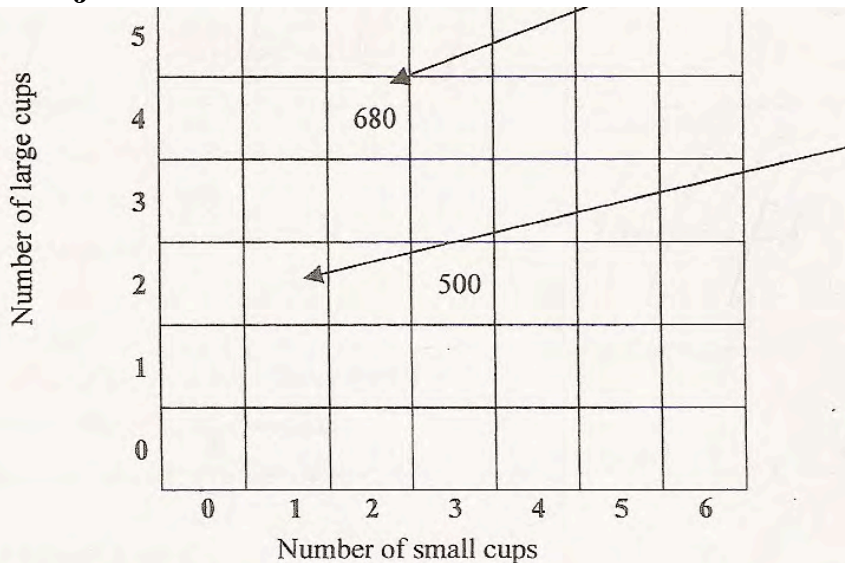
1. Explain what the number 500 in the chart means.

2 small cups and 3 large cups cost ^{X} 0

500 ¢

2. Use the information in the chart to find the cost of a small cup of coffee and the cost of a large cup of coffee. Show how you figured it out.

Small cup of coffee costs __2__ cents   Large cup of coffee costs __3__ cents $^X 0$

I Figured by looking at the graph and solving it.

3. What number should go in the empty box that the arrow is pointing to. __The number__
Explain your work.
that should go there is 580 because, $^X 0$
it is less on the graph.

Algebra   **Task 5**   **Coffee**

Student Task	Use a chart to solve simultaneous equations.
Core Idea 3 Algebraic Properties and Representations	**Represent and analyze mathematical situations and structures using algebraic symbols.** • Write equivalent forms of equations, inequalities and systems of equations and solve them. • Understand the meaning of equivalent forms of expressions, equations, inequalities, or relations.

Mathematics of this task:
- Read and interpret a diagram
- Write equations from a graphical representation
- Solve 2 equations with 2 unknowns to find a unit price
- Use proportional reasoning or equations to find the cost of an amount not on the table

Based on teacher observations, this is what algebra students knew and were able to do:
- Read and interpret the table to describe the 500 using appropriate units
- Find the cost in part 2
- Use guess and check to find the unit prices in part 2

Areas of difficulty for algebra students:
- Using the information in the table to write equations
- Solving simultaneous equations or recognizing the using equations could be a strategy to find the unit prices
- Trying to impose a unit of scale on the table without considering all the information
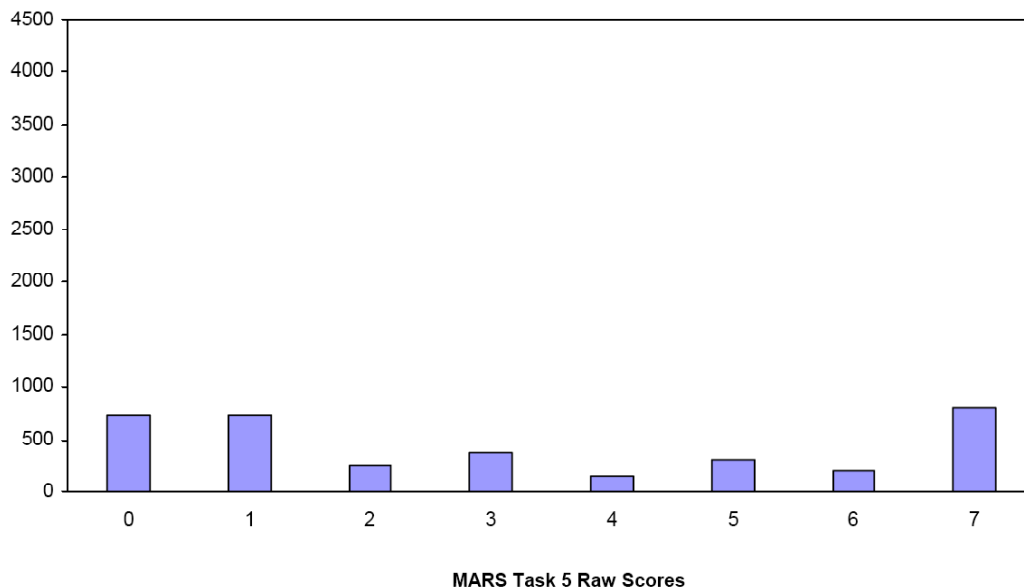
Task 5 - Coffee

Mean: 3.19   StdDev: 2.69

Table 49: Frequency Distribution of MARS Test Task 5, Course 1

Task 5 Scores	Student Count	% at or below	% at or above
0	724	20.6%	100.0%
1	728	41.2%	79.4%
2	251	48.4%	58.8%
3	368	58.8%	51.6%
4	147	63.0%	41.2%
5	310	71.8%	37.0%
6	200	77.5%	28.2%
7	793	100.0%	22.5%

Figure 58: Bar Graph of MARS Test Task 5 Raw Scores, Course 1

MARS Task 5 Raw Scores

The maximum score available on this task is 7 points.
The minimum score needed for a level 3 response, meeting standards, is 4 points.

Many students, 79%, were able to describe the meaning of the 500 in the table using appropriate units. More than half the students could find the cost of two large and one small coffee using proportional reasoning or values from their work in part 2. Almost half, 41%, could describe the 500 in the table, find the unit prices in part 2, and showed some algebra to find the unit price. 22% of the students met all the demands of the task including solving simultaneous equations to find the unit price. 20% of the students scored no points on this task. 93% of the students with the students with this score attempted the task.

Algebra

100

Coffee

Points	Understandings	Misunderstandings
0	93% of the students with this score attempted the task.	Students had difficulty describing the 500. 13% put price in cents but did not describe what was purchased. 10% read the number of coffees incorrectly from the scale.
1	Students could describe the meaning of the 500 in the table.	Students had difficulty finding the cost represented by the empty box. 10% of the students did not attempt this part of the task. 15% gave answers with no supporting work. Other answers vary because of responses in part 2.
3	Students could describe the 500 and find the value represented in the empty box in the table.	Students struggled with finding the unit prices in part 2. 13% did not attempt part 2 or did work but never arrived at an answer. 10% showed no work on part 2.
4	Students could describe the 500, do some work in part 2 and use the result to answer part 3.	Students struggled with using algebra to find the unit prices. Most students used guess and check. Some students attempted to use estimation from the other values in the table. Some students ignored one of the two scales when attempting to find the scale value. Some students made or used an incorrect equation, such as $x + y = 6$. Some put the information into 1 equation, such as $3x + 2y - 550 = 2x + 4y - 680$, which makes it impossible to eliminate one of the variables and solve.
7	Students could read and interpret a value in the table, use proportional reasoning or substitution to find the value of an empty box in the table, write equations from the information in the table, and solve the simultaneous equations to find the unit prices for large and small coffees.	

Implications for Instruction

Too often students learn short cuts that allow them to get correct answers relying on their arithmetic skills without learning the higher mathematics. They are rewarded for solutions rather than for progressing in their mathematical thinking. Students need to see good reasons to make the effort to leave behind what is comfortable to learn new skills and ideas. Students need to see reasons for letting go of guess and check and applying their algebraic skills to solving problems. Students need to have frequent opportunities to apply their skills in context to see the usefulness of the procedures being practiced in the textbook. It does a student no good to know a particular procedure, if the student doesn't know when to apply it in a practical situation.

Students didn't know how to use equations or see a purpose for writing equations to help them find the unit price in this task. Students who attempted to write equations either used only one, which didn't help to eliminate a variable, or combined unlike terms to get an equation that was untrue for their second equation. Students have not developed "algebraic" instincts about what type of information or algebraic equations would help them in a problem-solving situation.

Ideas for Action Research

A model for designing a lesson to help students let go of calculating an making the effort to learn new mathematics can be seen in the eighth grade TIMMS videos, Japanese Lesson 2: Algebraic Inequalities. This would be a good video to view with colleagues and then discuss how the principles might be applied in designing a follow up lesson for this task.

The lesson starts with the teacher posing a problem for the class. The teachers starts by saying that while they have been working on calculations for the last few days, today the lesson is about thinking. So the lesson today will be a bit more challenging.

The problem: *You need to buy 10 cakes for dinner. Some cakes cost 230 yen. Other cakes cost 200 yen. You cannot spend more than 2100.*
The teacher then talks about the context to make sure everyone understands the context. "Which cakes are most tasty? Which cakes do you want to buy the most of? How many do you need to buy in all?" So the purpose is to find out what is the most tasty cakes that you can buy and stay within you budget, making sure that everyone in the family gets a cake.

Students work the task and then prepare for class discussion. Most students use a guess and check method. So this is the solution shared first. The student describes the process. "First, I found out the cost of 10 tasty cakes, but that was too much. Then I found the cost of 9 tasty cakes which was lower than 2100 yen, but I needed to add 1 cake at 200 yen. So that cost too much. Then I tried 8 cakes. I would need to keep reducing by 1 tasty cake until I got a solution that would work. I didn't get a final solution, because I ran out of time. The teacher has probably purposively chosen someone with a slow method to show the inefficiency of this method.

Next the teacher and another student share more efficient strategies for solving the problem. The teacher explains a method examining differences. First the cost of 10 tasty cakes is 2300 yen, which is over by 200 yen. The difference between tasty cakes and the other cakes is only 30 yen. So, obviously more than one cake of the cheaper price will need to be purchased to

cover the 200 yen gap. Students figure out that the gap will need 7 of the cheaper cakes to make up the difference.

A student in the class then shares a strategy using an inequality. The student first sets the tasty cakes = x. So the amount of the other cake would be (10 – x). The inequality would then be
$230x + 200(x-10) \leq 2100$. The class then discusses the solution.

The purpose of letting the class solve the problem with their familiar solution strategy of guess and check first is to help them become grounded in the context and meaning of the solution. They are then ready to process other strategies and compare how the methods are similar.

The teacher now makes the unusual move, from our cultural perspective, of discussing the good qualities of the student's solution. The teacher points out using guess and check you need to do a lot of calculation, but with this method, using an inequality, the answer comes out quickly.

Finally, the teacher poses a follow up question with the prompt to "try on" the new strategy. This encourages students to make an effort and see a purpose in using the new mathematics.

Performance Assessment Task
Coffee Grade 10
This task challenges a student to represent a context by constructing two equations from a table. A student must be able to solve two equations with two unknowns and interpret the results in terms of the context of the situation including identifying the correct monetary units.
Common Core State Standards Math - Content Standards
High School – Functions – Building Functions **Build a function that models a relationship between two quantities.** F- BF.1 Write a function that describes a relationship between two quantities.   a.   Determine an explicit expression, a recursive process, or steps for calculation from a context. **High School – Algebra – Reasoning with Equations and Inequalities** **Solve systems of equations** A-REI.6 Solve systems of linear equations exactly and approximately (e.g. using graphs), focusing on pairs of linear equations in two variables.
Common Core State Standards Math – Standards of Mathematical Practice
MP.2 Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize* – to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents – and the ability to *contextualize,* to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. **MP.4 Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
Assessment Results
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
10	2009	7	4	41%