
Decimals

This problem gives you the chance to:

- show understanding of decimal numbers
-

1. Write each of this set of numbers in the correct box.

The box on the left is for numbers smaller than 5.5.

The box on the right is for numbers bigger than 5.5.

5.7 5.35 5.025 5.9 5.24 5.473

The first one has been done for you.

Numbers smaller than 5.5

Numbers bigger than 5.5

5.7

2. Which number is nearest to 5.5?

Explain how you figured this out.

3. Write down a number of your own that is bigger than 5.24 and smaller than 5.35.

4. Write the numbers in order from smallest to largest.

Explain how you decided which was the smallest number.

Decimals	Rubric	
<p>The core elements of performance required by this task are:</p> <ul style="list-style-type: none"> • show understanding of decimal numbers <p>Based on these, credit for specific aspects of performance should be assigned as follows</p>	points	section points
<p>1. Gives correct answers:</p> <p>In the left hand box: 5.35 5.025 5.24 5.473</p> <p>In the right hand box: (5.7), 5.9</p> <p><i>Partial credit</i></p> <p>Lose one point for each number incorrectly placed</p>	<p>3</p> <p>(2)</p> <p>(1)</p>	<p>3</p>
<p>2. Gives correct answer: 5.473</p> <p>Gives a correct explanation such as: It rounds to 5.5 and no other number does.</p>	<p>1</p> <p>1</p>	<p>2</p>
<p>3. Gives correct answer: any number larger than 5.24 and less than 5.35</p>	<p>1</p>	<p>1</p>
<p>4. Gives correct answer: 5.025 5.24 5.35 5.473 5.7 5.9</p> <p><i>Partial credit</i></p> <p>One error or First and last correct.</p> <p>Gives a correct explanation such as: I looked at the first number after the decimal point and choose zero because it was smallest.</p>	<p>2</p> <p>(1)</p> <p>1</p>	<p>3</p>
Total Points		9

Decimals

Work the task and look at the rubric. What are the big mathematical ideas being assessed in this task? _____

Students seemed to have difficulty comparing numbers with different numbers of digits. This is easiest to see by looking at how students ordered numbers in part 4. How many of your students were able to:

- Order all of the numbers correctly? _____
- Ordered most numbers correctly (1 or 2 numbers misplaced)? _____
- Order all the numbers with tenths, then the numbers with hundredths, then the numbers with thousandths?
- Ordered the wrong set of numbers or made up their own numbers? _____

Why do you think students have difficulty with this idea? What are some of their underlying beliefs about digits and size of numbers? What kinds of activities will help them confront those fallacies explicitly?

Now look at how students compare numbers in part 2. How many of your students put:

5.473	5.7	5.35	5.025	Other

List some of the rationales students made for their comparison.

How many students used the idea of comparison subtraction? _____

Did students attempt comparison of more than one choice to determine the optimal solution? _____

How do we help students to develop the habit of mind of comparing more than one possibility before making a decision?

Now look at explanations in part 4, how did you determine the smallest number. List some examples that showed a good understanding of place value.

What were some of the explanations that showed problems with understanding place value?

Looking at Student Work on Decimals

Student A is able to meet all the demands of the task. Notice that the student understands, but has difficulty with the language, that the tenths need to be compared before hundredths. *What do you think the student means by there is nothing in the tenths place? What do you want students to understand about zero in the tenths place? How often do students get opportunities to use language around place value? How does conversation and developing academic vocabulary expand the ways students think mathematically?*

Student A

The box on the right is for numbers bigger than 5.5.

~~5.7~~ ~~5.35~~ ~~5.025~~ ~~5.9~~ ~~5.24~~ ~~5.473~~

The first one has been done for you.

Numbers smaller than 5.5

5.35 ✓
5.025 ✓
5.24 ✓
5.473 ✓

Numbers bigger than 5.5

5.7
5.9 ✓

33

2. Which number is nearest to 5.5?
Explain how you figured this out.

5.473 ✓ 11

How I figured this out is that I rounded the tenths place for each number and saw that 5.473 was closest. 5.473 rounds 5.5. 11

3. Write down a number of your own that is bigger than 5.24 and smaller than 5.35.

5.3 ✓ 11

4. Write the numbers in order from smallest to largest.

5.025, 5.24, 5.35, 5.473, 5.7, 5.9 22

Explain how you decided which was the smallest number.

I knew that 5.025 was the smallest because it is in the hundredths and not the tenths place. The rest of the numbers besides 5.025 have numbers in the tenths place. Also tenths place is bigger. 9

Students really struggled with the thinking needed to explain why a number is smallest. See the work of Student B.

Student B

4. Write the numbers in order from smallest to largest.

5.025, 5.24, 5.35, 5.473, 5.7, 5.9 ✓ 2 2

Explain how you decided which was the smallest number.

It was the closest to five and 0 all of the numbers have a whole number that is five.

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Student C is starting to make comparisons between numbers. In part 2 the student compares two options before making a choice. In part 4 the student chooses between two choices with digits in the thousandths, but doesn't seem to be able to articulate how to compare numbers with different numbers of digits. This is interesting because the student was able to order all the numbers. *Again, how do we help students develop the ability to think with academic language and use it to make sense of our choices?*

Student C

The first one has been done for you.

Numbers smaller than 5.5

5.35 ✓ 5.24 ✓
5.02 ✓ 5.473 ✓

Numbers bigger than 5.5

5.7 ✓ 5.9 ✓
5.473 ✓
5.35 X X

2. Which number is nearest to 5.5? Explain how you figured this out.

Because 5.35 is only 0.15 away from 5.5 or 5.50 and 5.7 or 5.70 is 20 away

3. Write down a number of your own that is bigger than 5.24 and smaller than 5.35.

5.28 ✓ 11

4. Write the numbers in order from smallest to largest.

5.025, 5.24, 5.35, 5.473, 5.7, 5.9 ✓ 2 2

Explain how you decided which was the smallest number.

I decided 5.025 was the smallest because it's twenty five thousandths and 5.473 I think is bigger than

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Student D compares numbers in the tenths when making a choice in part 2. In part 4 the student understands which number is smaller and can give additional information about the number by writing it as a fraction. But, the student does not have the idea of making a comparison between the whole set of numbers.

Student D

Numbers smaller than 5.5

5.025 ✓
 5.35 ✓
 5.24 ✓
 5.473 ✓

Numbers bigger than 5.5

5.9 ✓
 5.7

3

3

2. Which number is nearest to 5.5?
 Explain how you figured this out.

5.7 X X 0

5 = 5 wholes, 7 = 7 tenths X X 0
5 = 5 wholes, 5 = 5 tenths

3. Write down a number of your own that is bigger than 5.24 and smaller than 5.35.

5.35 X X 0

4. Write the numbers in order from smallest to largest.

5.025, 5.24, 5.35, 5.473, 5.7, 5.9 ✓ 2

Explain how you decided which was the smallest number.

5.025 = 5 ²⁵/₁₀₀₀ X X 0

Student E is able to correctly choose the number closest to 5.5, but has no supporting logic. Its closest because it closest. In part 4 the student orders the wrong set of numbers, but the logic is still the same. It is smallest because its smallest. *How do we help students understand the logic of making a convincing argument?*

Student E

THE FIRST ONE HAS BEEN DONE FOR YOU.

Numbers smaller than 5.5	Numbers bigger than 5.5
5.35 ✓ 5.025 ✓ 5.24 ✓ 5.473 ✓	5.7 5.9 ✓

2. Which number is nearest to 5.5?
 Explain how you figured this out.

3
3
1

✓ I saw that five point 4.73 is the nearest because its closer to 5.5. 1
0

3. Write down a number of your own that is bigger than 5.24 and smaller than 5.35.

1
1

5.26 ✓

4. Write the numbers in order from smallest to largest.

0
0

$5.24, 5.26, 5.35$ A X

Explain how you decided which was the smallest number.

X I saw that 24 is smaller and 26 is the medium number and 35 is the largest. X 0

9

Student F has some vocabulary to indicate distance in part 2, but doesn't quantify the distance. The student only considers the one possibility. In part 4 the student makes up her own set of numbers to order. Notice the student doesn't understand the two of the choices are equal.

Student E

The first one has been done for you.

Numbers smaller than 5.5	Numbers bigger than 5.5
<p>5.025 ✓✓ 5.35 ✓✓ 5.24 ✓✓ 5.473 ✓✓</p>	<p>5.7 5.9 ✓✓</p>

3

2. Which number is nearest to 5.5?
Explain how you figured this out.

5.7 x x 0

Because it is to less away from 0

5.5. x x 0

3. Write down a number of your own that is bigger than 5.24 and smaller than 5.35.

5.29 ✓✓ | |

4. Write the numbers in order from smallest to largest.

5.2, 5.20, 5.25, 5.28, 5.29 x 0

Explain how you decided which was the smallest number.

Because it is lower than 5.29 x 0

Student F has a more limited flexibility in working with place value. While the student can realize that the 5.025 is smaller than 5.7, the student doesn't recognize that 5.473 is also smaller than 5.7. The language in part 2 does not indicate measuring the distance between numbers and 5.5 to make the comparison. In part 4 the student has chosen only some of the numbers to order. *What experiences do you think this student might need?*

Student F

Numbers smaller than 5.5

5.24 ✓✓	5.35 ✓✓
5.025 ✓✓	

Numbers bigger than 5.5

5.7	5.9 ✓✓
	5.473 X X

(2) (2)

2. Which number is nearest to 5.5?
Explain how you figured this out.

5.35 X X O O

I think that because it's ~~100%~~ bigger. X O

3. Write down a number of your own that is bigger than 5.24 and smaller than 5.35.

5.30 ✓✓ | |

4. Write the numbers in order from smallest to largest.

5.24^X, 5.025^X, 5.35^X, 5.5, 5.7, 5.9 O O

Explain how you decided which was the smallest number.

I decided that 5.24 is the smallest + X O O

because there is no other one that is small. 9

Look at the work of Student G. What strategy do you think the student uses to order the numbers? What is the misconception?

Student G

Numbers smaller than 5.5

x x

Numbers bigger than 5.5

✓ 5.7 x 5.9 x 5.35 x 5.025
x 5.24 x 5.473

2. Which number is nearest to 5.5?
Explain how you figured this out.

5.7 x 1 0

5.7 is only two ahead 5.50 x 0

3. Write down a number of your own that is bigger than 5.24 and smaller than 5.35.

5.30 ✓ 1

4. Write the numbers in order from smallest to largest.

5.7 5.9 5.24 5.35 5.025 5.473 x x 0 0

Explain how you decided which was the smallest number.

Every number in there is x 0
bigger than 5.7, x 0

9

Student Task	Show understanding of decimal numbers.
Core Idea 2 Number Properties	<p>Understand numbers, ways of representing numbers, relationships among numbers, and number systems.</p> <ul style="list-style-type: none"> • Recognize and generate equivalent forms of commonly used fractions and decimals. • Understand the place-value structure of the base-ten number system including being able to represent and compare rational numbers.

Mathematics of the task:

- Comparing decimal numbers with different numbers of digits
- Understanding that comparison involves looking at all the options
- Understanding zero as a place value
- Making a mathematical justification
- Being able to use place value language in context to make explanations and convincing arguments

Based on teacher observations, this is what fifth graders knew and were able to do:

- Order decimals with the same number of digits and find numbers between
- Sort some of the numbers in relation to the 5.7

Areas of difficulty for fifth graders:

- Making a comparison after considering all the alternatives
- Using academic language in context
- Making comparison between decimals with different numbers of digits

MARS Test Task 5 Frequency Distribution and Bar Graph, Grade 5

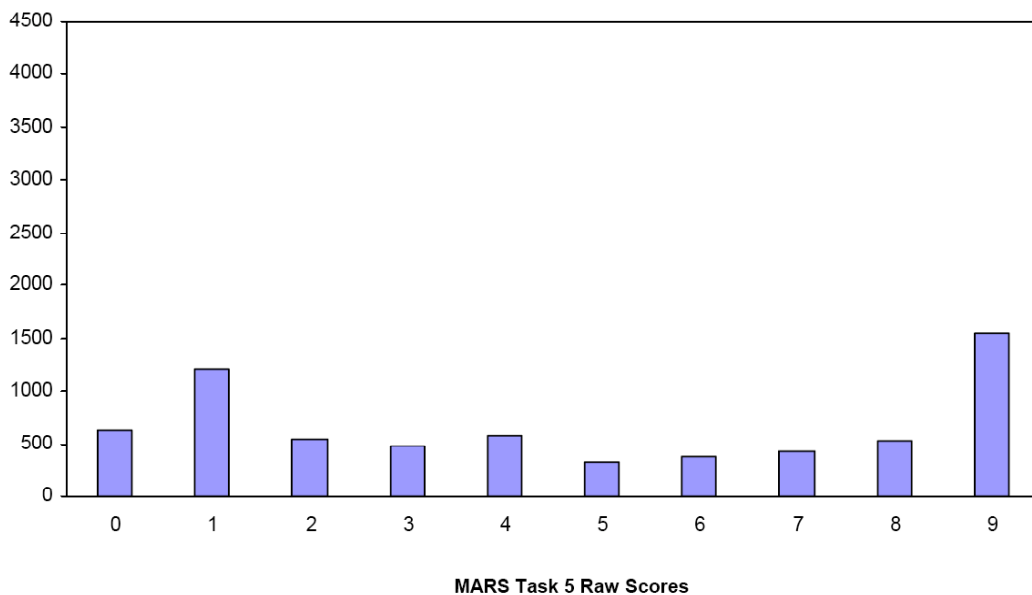
Task 5 - Decimals

Mean: 4.68 StdDev: 3.33

Table 29: Frequency Distribution of MARS Test Task 5, Grade 5

Task 5 Scores	Student Count	% at or below	% at or above
0	628	9.5%	100.0%
1	1205	27.8%	90.5%
2	539	35.9%	72.2%
3	473	43.1%	64.1%
4	572	51.8%	56.9%
5	316	56.6%	48.2%
6	372	62.2%	43.4%
7	431	68.7%	37.8%
8	526	76.7%	31.3%
9	1537	100.0%	23.3%

Figure 38: Bar Graph of MARS Test Task 5 Raw Scores, Grade 5



The maximum score available for this task is 9 points.

The minimum score needed for a level 3 response, meeting standards, is 4 points.

Most students, 90%, could find a number between two values with the same number of digits. More than half the students, 57%, could also sort the numbers into the two boxes (greater than 5.5 and smaller than 5.5). Some students, about 38%, could also order the decimals and choose the smallest one. About 23% were able to meet all the demands of the task including comparing the numbers and choosing the one closest to 5.5. Almost 10% of the students scored no points on this task. All of the students in the sample with this score attempted the task.

Decimals

Points	Understandings	Misunderstandings
0	All the students in the sample with this score attempted the task.	Students had difficulty finding a number between 5.24 and 5.35.
1	Students could find a number between two decimal values in the hundredths.	Students had difficulty sorting the numbers into more than 5.5 and less than 5.5. 15% put all the numbers in the box for greater than 5.5. 8% put all the numbers in the box for less than 5.5. 18% misplaced 5.473. 12% misplaced 5.24.
4	Students could sort the values between less than 5.5 and greater than 5.5 and find values between 5.24 and 5.35.	Students had difficult ordering the numbers from smallest to largest and explaining how to identify the smallest number. 37% ordered by number of digits (5.7, 5.9, 5.24, 5.35, 5.025, 5.473).
7	Students could sort and order all the values, identify the smallest value, and locate values between 5.24 and 5.35.	Students had difficulty finding the value closest to 5.5. Students often only considered one or two numbers before making a choice. 56% thought the closest number was 5.7. 9% thought the closest value was 5.35.
9	Students understood decimal place value and could compare values of numbers with different amount of digits after the decimal point.	

Implications for Instruction

While students could order and compare numbers with the same number of digits after the decimal, students struggled with ordering and comparing numbers of differing amounts of digits after the decimal. While it is convenient for reading answers to say 5 point 24, this use of language takes the concept of “value” out of what is heard and communicated. Although it may at first be confusing to some students that use of mathematical language in decimals is critical for building a meaning and sense of value.

Learning to “go around” significant numbers is an important step for beginning counters. What comes after 58, 59 ... or 98, 99, ... When students are learning place value, they need to go through some of those same counting strategies to make sense of value. Activities, such as finding what is $\frac{1}{10}$ more than 3.6 or 4.87 or what is $\frac{3}{100}$ less than 5.78 helps students develop this sense of order.

Students should have frequent opportunities to locate values on a number line to help them recognize relative values. This helps them see the difference between numbers like 5.473 and 5.7 without learning or remembering procedures like adding zeroes to make all the numbers the same size.

Students need a variety of activities, both verbal and written, to help them make sense of numbers with a different amount of digits after the decimal.

Ideas for Action Research

Plan some lessons or warm ups to help students become more fluent with place value meaning. Start with some tasks of adding a constant amount (similar to skip counting at earlier grade levels). For example, give students a short series of problems, such as:

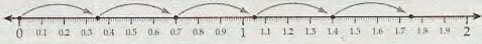
1. 0.2, 0.4, 0.6, _____, _____, _____, _____ (add on 0.2 each time)
2. 0.25, _____, _____, _____ (add 0.25 each time)
3. 0.05, _____, _____, _____ (add 0.05 each time)

Don't worry about mistakes at this time. The purpose of the activity is to let students self-correct and confront their misunderstandings. When the students have finished give them number lines where they can mark the growth on the number and see their errors. Then have them reconfirm the answers using a calculator.

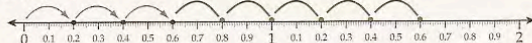
CHECK Here students check their decimal sequences in two ways: using a number line and using the calculator. If students are unclear how to perform these checks, work through the first problem with them as an example.

CHECK Check your answers by skip counting on the number line and by using a calculator (for example, press $.35 + .35 = __ + .35 = ____$)

Example
Skip by 0.35.

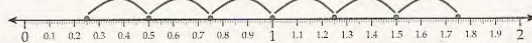


1. a. Skip by 0.2.



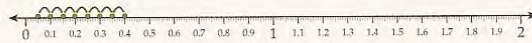
b. Add on 0.2 using your calculator: 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6

2. a. Skip by 0.25.



b. Add on 0.25 using your calculator: 0.25, 0.50, 0.75, 1.00, 1.25, 1.50, 1.75

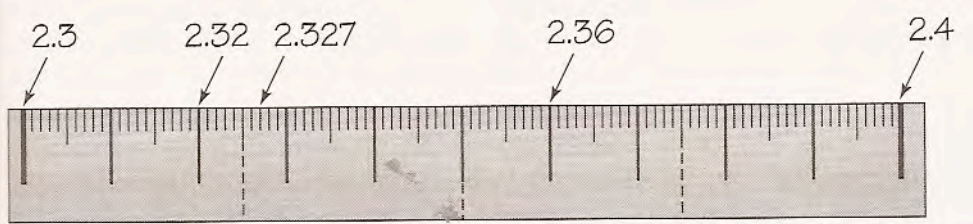
3. a. Skip by 0.05.



b. Add on 0.05 using your calculator:
0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40

Example taken from *Mathematics Navigator, Decimals and Powers of Ten* published by America's Choice.

Another simple lesson is to make a list of four or five decimal numbers that students might have difficulty putting in order, such as the number set from the task. Ask students to predict the order of the numbers from least to greatest. After everyone has made their predictions, put a large number line on the white board at the front of the class. Now use this as to have students discuss the precise locations of the numbers. By making a blank number line, the endpoints can be changed to make new problems.



Cut four strips of poster board measuring 6×28 inches. Tape end to end. Place on chalk tray.
Write on board above. Endpoints can be any interval of $1, \frac{1}{10}, \frac{1}{100}$.

Example taken from *Teaching Student-Centered Mathematics, Grades 5-8* by John Van de Walle.

A final activity might be to look at the concept, "close to". Start with a four-digit decimal on the board, such as 4.1736. To get students thinking, first ask them to consider if the number is closer to 4 or 5. Then slowly think about narrowing in on the place values. Is it closer to 4.1 or 4.2? In conjunction with this type of warm up, it might also be good to ask them to compare. How close is it to 4.2? to 4.15? This helps students to really locate the number relative to a variety of landmarks.

Performance Assessment Task				
Decimals Grade 5				
This task challenges a student to use knowledge of place value system to represent and compare rational numbers. A student must use place value understanding to justify or explain how to order a set of decimal values. A student must be able to make sense of rational values to generate numbers falling between two values.				
Common Core State Standards Math - Content Standards				
<p>Number and Operations in Base Ten Understand the place value system.</p> <p>5.NBT.3 Read, write, and compare decimals to thousandths.</p> <ol style="list-style-type: none"> Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g. $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) = 9 \times (1/100) = +2 \times (1/1000)$. Compare two decimals to thousandths based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons. <p>5.NBT.4 Use place value understanding to round decimals to any place value.</p>				
Common Core State Standards Math – Standards of Mathematical Practice				
<p>MP.3 Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even through they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others,</p> <p>MP.6 Attend to precision. Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.</p>				
Assessment Results				
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.				
Grade Level	Year	Total Points	Core Points	% At Standard
5	2009	9	4	56.9%