Half and Half

These shapes show halves. One half of the shape is colored and one half of the shape is uncolored.



(Yes) No   (Yes) No   (Yes) No

2 + **2** =**4**   **2**+ **2** = **4**   **2** + **2** = **4**

These shapes do **not** show halves.

Yes (No)   Yes (No)

3 + **2** = **5**   **3** + **2** = **5**

Look at the three shapes below and on the next page. Find out if you can color in one half of the shape. Circle "yes" or "no" below the shape. Then, write an addition sentence that describes how you colored in the shape.

1.)   2.)

Yes   No   Yes   No

___ +___= ___   ___ + ___= ___

3.)

Yes   No

____ + ____ = ____

4. How can you tell by looking at a shape that it can be colored to show half?

_____

_____

_____

_____

Half and Half Grade 2		Rubric	

The core elements of the performance required by this task are:
   • Uses number concepts in geometric contexts
   • Represents commonly used fractions such as ½
 	
Based on these credit for specific aspects of performance should be assigned as follows
		points	section points

1	Gives correct answer as: Correct drawing that shows halves **Yes** **4 + 4 = 8**	**1** **1** **1**	 **3**
2	Gives correct answer as: Drawing that shows inequality **No** Equation matches drawing Alternative solution: Divides the figure in half using fractions of a square **Yes** **4 ½ + 4 ½ = 9**	**1** **1** **1** **or** **1** **1** **1**	 **3**
3	Gives correct answer as: Correct drawing that shows halves **Yes** **6 + 6 = 12**	**1** **1** **1**	 **3**
4	Explains how they got the answer such as: The number sentence is always a double. There are even numbers of squares so you can find half and half. Each colored square has a partner that is uncolored.	 **1**	 **1**
	Total Points		**10**

Looking At Student Work – Half and Half:

Many of the students, 24%, were able to meet all the demands of this task. The final question on this task asked students to explain how they knew when a shape could have its parts divided in half. Below are four examples of their thinking. Student A not only explains that all shapes with an even number of small squares would be a "yes" but also that a shape with an odd number of small squares would be a "no".

Student A

Half and Half

These shapes show halves. One half of the shape is colored and one half of the shape is uncolored.
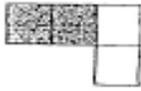
(Yes) No

$2 + 2 = 4$

(Yes) No

$2 + 2 = 4$
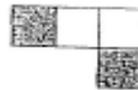
(Yes) No

$2 + 2 = 4$

These shapes do **not** show halves.
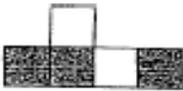
Yes (No)

$3 + 2 = 5$
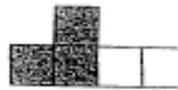
Yes (No)

$3 + 2 = 5$

Look at the three shapes below and on the next page. Find out if you can color in one half of the shape. Circle "yes" or "no" below the shape. Then, write an addition sentence that describes how you colored in the shape.

1.)

(Yes)   No   ✓   3

$4 + 4 = 8$ ✓

2.)

Yes   (No) ✓   3

$5 + 4 = 9$ ✓

3.)

Yes   No   ✓   3
✓
6 + 6 = 12   ✓

1. How can you tell by looking at a shape that it can be colored to show half?

It's because the "yes"'s are
even numbers and the "no's are
odd numbers.   ✗

Student B's explanation is an example of those students who spoke to each "side" or half needing to be equal in order to show half.

Student B

3.)
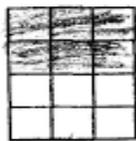
Yes ✓   No ✓
6 + 6 = 12

4. How can you tell by looking at a shape that it can be colored to show half?

if each side is equal then it
shows half,

While Student C's explanation is not completely clear, it is evident that the student knows that the colored half and the uncolored half must have the same number of squares.

Student C
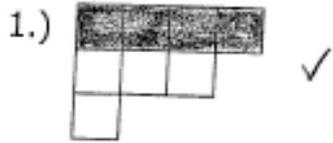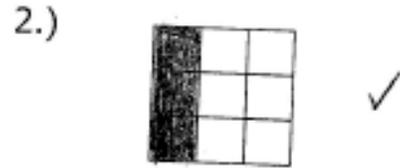
1.)

 ✓

Yes (circled)   No ✓

$\underline{4} + \underline{4} = \underline{8}$ ✓

2.)

 ✓

Yes   No (circled) ✓

$\underline{3} + \underline{6} = \underline{9}$ ✓

3.)

 ✓

Yes (circled)   No ✓

$\underline{6} + \underline{6} = \underline{12}$ ✓

4. How can you tell by looking at a shape that it can be colored to show half?

You can tell by looking at a shape that can be colored half because it has to have the same number as the one you color.

Student D explained how to show half by citing the specific case of part 3, 6 + 6 = 12. Again, this explanation references that they same number are "colored in" as are left "plain white" in order to show half.

Student D

3.)

Yes   No

6 + 6 = 12

4. How can you tell by looking at a shape that it can be colored to show half?

If 6 squares are colored
and 6 squares are plaian white
the it is half.

Many students were vague or incomplete in their explanations for how to tell if a shape can be colored in half. Students E and F explain procedures for coloring or drawing but do not address when it can be colored in halves.

Student E

3.)
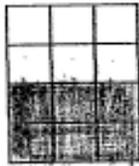
Yes ✓   No

6 + 6 = 12 ✓

4. How can you tell by looking at a shape that it can be colored to show half?

I can show by looking at it and
by counting it.

Student F

3.)

(Yes) ✓   No

___6___ + ___6___ = _12_ ✓

---

4. How can you tell by looking at a shape that it can be colored to show half?

You could make ^x half by doing it with liker cubes. And drawing it.

Student G explains how a specific shape can be colored in half and why. She also knows that shape 2 cannot be colored in half the way it is. Unfortunately, she did not give a number sentence that reflected the way in which she colored shape 2.

Student G
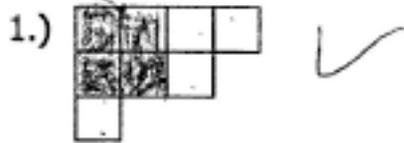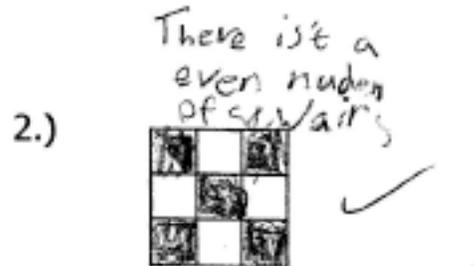
1.)

There is a even number of squairs

2.)

Yes   No ✓

$4 + 4 = 8$ ✓

Yes   No ✓

___ + ___ = ___ ✗

3.)

Yes   No

$6 + 6 = 12$

4. How can you tell by looking at a shape that it can be colored to show half?

because teys 6 and 6 are 12. and 12 is a even number.

Students H, I, and J can clearly explain the reasoning for why a shape like these could be colored in half. Student H sees the relationship between even numbers and finding half but did not find half of 12 for part 3.

Student H

3.)
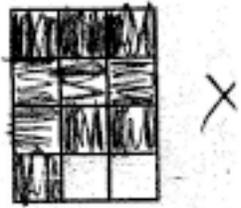
Yes   (No)

$2 + 10 = 12$

4. How can you tell by looking at a shape that it can be colored to show half?

A shape can be half only if it is even. ✓

Student I's paper is reflective of many students work. Many felt the need to find a vertical symmetrical line in the shape that would allow the shape to be divided into half. This need was so strong that it allowed for the equations and drawings to reflect both different ideas.

Student I
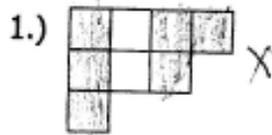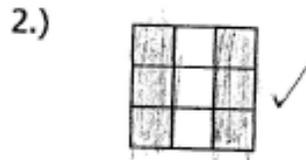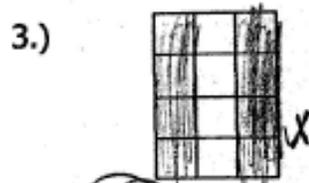
1.)

Yes ✓   No

$4 + 4 = 8$ ✓

2.)

Yes   No ✓

$5 + 4 = 9$ ✗

3.)

Yes ✓   No

$6 + 6 = 12$

4. How can you tell by looking at a shape that it can be colored to show half?

You can tell by draw Sqars inside count how many Sqars there are if its an even number the answer is yes and if it an odd number the answer is no,

Student J is able to explain when making half would work and when it would not work but his attempts to show this in the first three parts were not reflective of this thinking.

Student J

1.)

$\times$

Yes   No ✓

$\underline{2} + \underline{2} = \underline{4}$ $\times$

2.)

✓

Yes   No ✓

$\underline{2} + \underline{1} = \underline{3}$ $\times$

3.)

$\times$
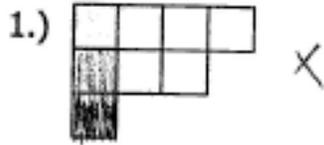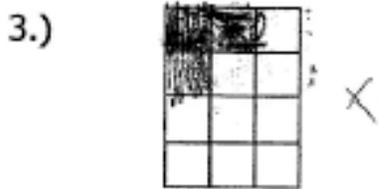
Yes   No ∨

$\underline{3} + \underline{3} = \underline{6}$

4. How can you tell by looking at a shape that it can be colored to show half?

I could tell because whenever the do yes it is even and when we do no it is an odd number.

Student K's paper also demonstrates the desire to find a vertical line of symmetry to cut the shape in half – at least for the 2nd and 3rd shape. The center line of squares was ignored.

Student K
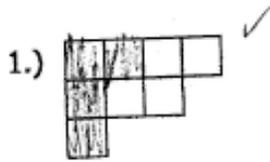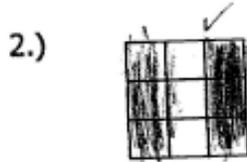
1.)

Yes   No

$4 + 4 = 8$

2.)

Yes   No

$3 + 3 = 6$

3.)

Yes   No

$4 \times 4 = 8$
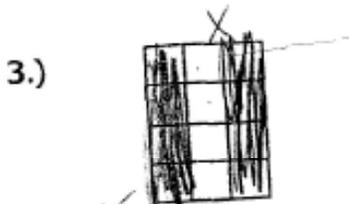
4. How can you tell by looking at a shape that it can be colored to show half?

Because I read the instructions. Als I knew I could do it.

Student L's demonstrates an understanding of two parts with equal numbers but the student does not pay attention to the uncolored squares left inside each shape.

Student L
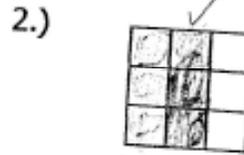
1.)

Yes   No

$3 + 3 = 6$

2.)

Yes   No

$3 + 3 = 6$

3.)

Yes   No

$4 + 4 = 8$
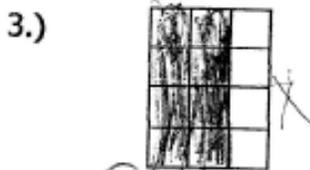
4. How can you tell by looking at a shape that it can be colored to show half?

I know why because I could tell if and even,

Student M erroneously determines that if the shape has doubles using odd numbers it can not be divided in half and if the shape has doubles using even numbers then it can be divided into half. Again, attention is paid to the colored part of each shape and not to the uncolored part of each shape.

Student M
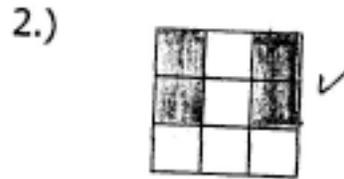
1.) ✗

Yes ✗ (No) ✗

$3 + 3 = 6$

2.) ✓

(Yes) ✗ No

$2 + 2 = 4$

3.)

(Yes) ✓ No

$4 + 4 = 8$

Teacher Notes:

_____

_____

_____

_____

Half and Half
Mean: 7.24   StdDev: 2.84

MARS Task 5 Raw Scores
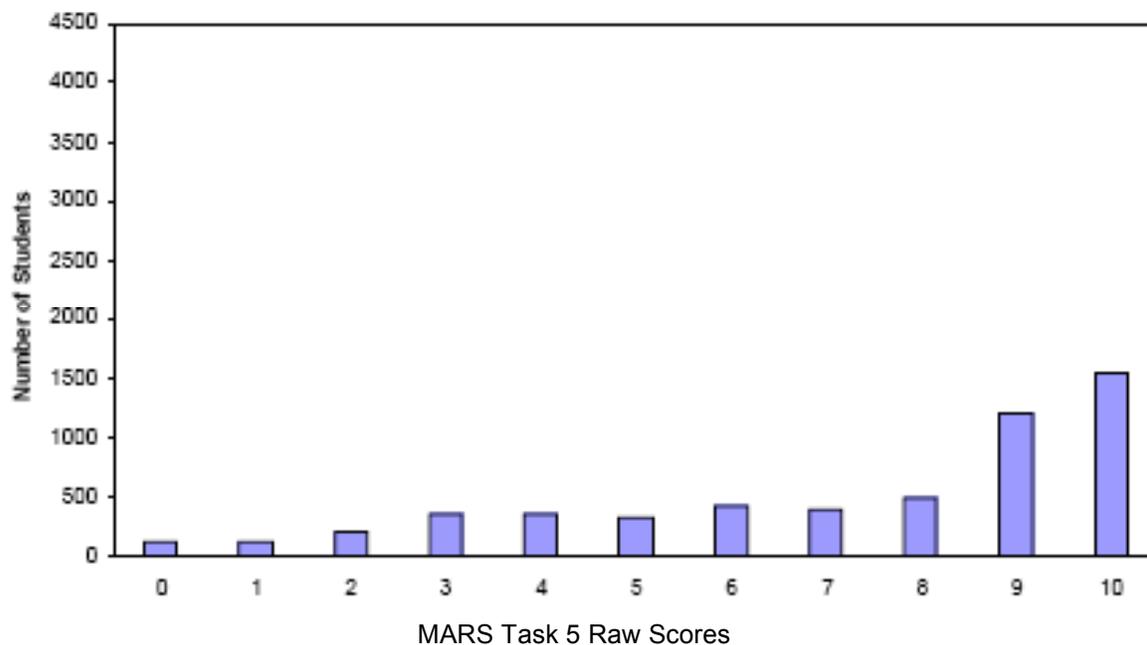
Score:	0	1	2	3	4	5	6	7	8	9	10
Student Count	114	122	211	360	363	319	428	399	493	1209	1542
% < =	2.1%	4.2%	8.0%	14.5%	21.0%	26.8%	34.5%	41.7%	50.5%	72.3%	100.0%
% > =	100.0%	97.9%	95.8%	92.0%	85.5%	79.0%	73.2%	65.5%	58.3%	49.5%	27.7%

The maximum score available for this task is 10 points.
The cut score for a level 3 response is a 5.

Many students, 74% could find a way to divide shape one and shape three into halves. 88% of the students knew that shape number two could not be colored in half as it was presented. Almost one half of the students, 56%, could give an explanation as to how to determine one half of a shape. Almost 28% of the students could meet all the demands of the task including explaining how to find one half of a shape. 79% of the students met the essential demands for the task. Only 2% of the students scored no points on this task. 50% of the students with a score of 0 attempted the task.

Half and Half

Points	Understandings	Misunderstandings
0 - 2	Less than 1% of the students did not attempt this task.	Little relationship was shown between a shape colored in half and a yes or no response. When a student colored some of the squares within the shape and left some uncolored, the colored shapes were paid attention to and the uncolored ones were ignored. Equations did not match the colored shapes.
3 - 4	Students with a score of 3 or 4 could answer a little less than half (4 out of 9) of the questions about the three shapes.	About half of these students did not attempt the explanation. Many of the student explanations were vague or incomplete or spoke to the procedure followed rather than to what concept made half of a shape.
5	Student explanations focused on the overall even number of squares needed to make half or having the same number of squares colored or uncolored.	Students considered finding a vertical line of symmetry necessary to find half. About half of these students struggled with shape 1 and 2 but found half in shape 3 and gave an explanation to support dividing that shape in half or any other shape in half.
6 - 7	These students were able to successful answer two-thirds or more of the questions about the three shapes. One half of these students saw a relationship between doubles and an answer of yes for dividing the squares in half.	Student explanations were vague and did not answer how to find half of a shape. Many times the explanation was about the steps one might take to look at and count the squares but did not address what constituted half of all the squares.
8 - 9	Most of the questions for the three shapes were answered correctly. Successful student explanations focused on evenness, doubles and/or having the same number colored as uncolored.	Students struggled with how to represent the 2nd shape. Some students chose to not color the shape at all. Many students did not write an equation. Others did not mark yes or no for question 2. In several cases, student explanations focused on the procedures of the task rather than on issues of finding half.
10	Students successfully answered all questions regarding the three shapes. The explanations of these students referenced the need for an even number of squares to in the shape, an equation of doubles and many times referenced not only when a shape could be divided into half but also when it could not be so divided.	

Based on teacher observations, this is what second grade students seemed to know and be able to do:

Areas of difficulty for second graders, students struggled with:
- problem 2 – students did not seem to understand how to work on problem 2
- were not able to explain how they know if it could be divided into half even if they could demonstrate so
- were incomplete in their explanation
- tried to divide by rows rather than by squares
- non-symmetrical shapes seemed to confuse some students

Strategies used by successful students:
- They understood the question and were not confused by how squares were arranged
- Could write equations to match their coloring
- Students used their knowledge of even and odd numbers and doubles to explain how to show half.
- Count and determine if a number is even and hence could be divided into half.
- Knew that having the same number colored as uncolored meant that they were coloring in half.

Questions for Reflection on Half and Half:
- What informal experiences do students have with fractions?
- What opportunities do students have to think about and talk about fractions in everyday situations?
- Do students have experiences in folding paper into halves and fourths and representing all or parts of the whole paper?
- What opportunities do students have to define fractions as equal sized portions and as "fair shares"?
- Do students use a variety of physical materials to represent fractions and fractional pieces?
- What opportunities do students have to see fractions represented as area models? Length or measurement models? As set models?
- What opportunities do students have for sharing their thinking?

Look at your student work, how many of your students:

Explained how to find one half of a shape.	Explained the procedure of the task but not how to find half.	Had a vague or incomplete explanation of how to find half.	Did not attempt explain how to find half.

- How do you build norms for classroom discussions that allow students to share strategies and justify their thinking?
- What kinds of questions can you ask students to find out if they are making sense of fractions?
- What kinds of activities or discussions might help students see relationships in equal sized portions or fair shares?

Teacher Notes:

Implications for Instruction:

Students need many and varied experiences to build their understandings around fractions. These experiences should include identifying and representing fractions as parts of a whole, parts of a group, and fractions on a number line. Fraction activities may be part of problems set in geometry and probability as well as in number contexts. This task, of determining whether or not a shape can be shaded in half, connects many mathematical concepts. Seeing that a shape with eight equal squares can be shaded with four squares and left un-shaded with another four squares is another way of showing that eight squares may be divided into two equal groups of four. This notion of equal groups first appears when young children learn the addition facts known as doubles: 1+1, 2+2, 3+3, and so on. The number sentence for the first and third shapes in this task, $4 + 4 - 8$ and $6 + 6 = 12$, come from the family of double facts. This task of covering a shape with equal numbers of shaded and un-shaded squares links addition, multiplication, division, and fractions. Many children showed a cursory understanding that shapes with an even number of squares could be divided into one half and that the shape with an odd number of squares could not. While the concept of odd and even numbers can be a difficult one for some children to grasp, with repeated experiences, in a variety of settings, children begin to discover that even numbers are those that can be divided into two equal sized groups, and odd numbers are those that cannot. Children can begin to construct their own understanding of odd and even numbers as well as their understanding of the fraction one-half. As children work with different ways to show halves, they have the chance to learn, demonstrate and apply some of the basic concepts of fractions: that a fraction is composed of a number of parts that are equal in size and that the fraction used to name a part of a shape does not depend upon the absolute size of the shape (i.e. one half can be 4 out of 8 squares as well as 6 out of 12 squares).

Teacher Notes:

Performance Assessment Task
Half and Half
Grade 2
The task challenges a student to demonstrate understanding of one-half in a geometric context. Students must be able to count the total number of same-size squares as well as the colored squares vs. non-colored squares. Students must make sense of these totals to determine if a shape shows halves or not. Students must be able to articulate how one can tell by looking at a rectangle partitioned into rows and columns of same-size squares whether or not this shape can be colored to show one-half.

Common Core State Standards Math - Content Standards
Geometry
Reason with shapes and their attributes.
2.G.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
2.G.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape

Common Core State Standards Math – Standards of Mathematical Practice
MP.3 Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
MP.8 Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through $(1, 2)$ with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1)$, $(x-1)(x2+x+1)$, and $(x-1)(x3 +x2+x+1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Assessment Results
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including

the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
2	2005	10	5	79 %