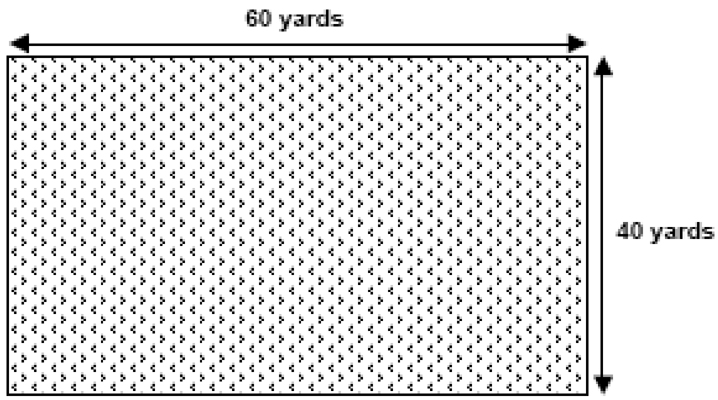
Lawn Mowing

This problem gives you the chance to:
• solve a practical problem involving ratios
• use proportional reasoning



Dan and Alan take turns cutting the grass.
Their lawn is 60 yards long and 40 yards wide.

1. What is the area of the yard? ______________ square yards

Dan takes an hour to cut the lawn using an old mower.

2. How many square yards does Dan cut in a minute? ______________
   Show your work.

Alan only takes 40 minutes using a new mower.

3. How many square yards does Alan cut in a minute? ______________
   Show your calculation.

4. One day they both cut the grass together.
   How long do they take? ______________
   Show how you figured it out.

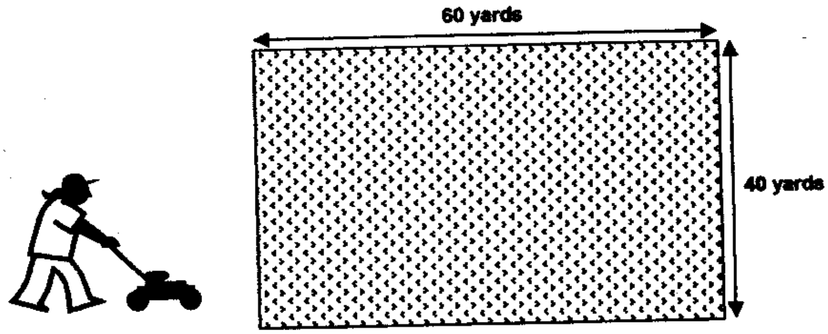
8

Lawn Mowing **Grade 7** Rubric		
The core elements of performance required by this task are: • solve a practical problem involving ratios • use proportional reasoning Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: **2,400** square yards	1	1
2. Gives correct answer: **40** square yards per minute Shows work such as: (60 x 40) ÷ 60	1 1ft	2
3. Gives correct answer: **60** square yards per minute Shows work such as: (60 x 40) ÷ 40	1 1ft	2
4. Gives correct answer: **24 minutes** Shows correct work such as: In one minute together they mow 40 + 60 = 100 square yards (60 x 40) ÷ 100	1 2ft	3
Total Points		**8**

Looking at Student Work on Lawn Mowing:

A good task allows students to choose a variety of tools to help solve the problem. Student A is able to use proportions to make sense of the rates in lawn mowing. The proportions really highlight that idea of rate by looking at the labels yards per minute, which is so important to understanding this situation. The proportion also allows the student to reason about combining the rates in part 4.

Student A



Dan and Alan take turns cutting the grass.
Their lawn is 60 yards long and 40 yards wide.

1. What is the area of the yard? $60 \times 40 = 2400$ — 2400 square yards

Dan takes an hour to cut the lawn using an old mower.

2. How many square yards does Dan cut in a minute? Show your work.
   $\frac{x}{1 \text{ min}} = \frac{2400 \text{ yards}}{60 \text{ min}}$ $x \cdot 60 = 1 \cdot 2400$ $\frac{60x}{60} = \frac{2400}{60}$ $x = 40$
   $60 \times 60 = 3600$
   $x =$ 40 yards in a min.

Alan only takes 40 minutes using a new mower.

3. How many square yards does Alan cut in a minute? Show your calculation.
   $\frac{x}{1 \text{ min}} = \frac{2400 \text{ yards}}{40 \text{ min}}$ $x \cdot 40 = 1 \cdot 2400$ $\frac{40x}{40} = \frac{2400}{40}$ $x = 60$
   $x =$ 60 yards/min.

4. One day they both cut the grass together. How long do they take? Show how you figured it out.
   60 yards + 40 yards = 100 yards
   $\frac{100 \text{ yards}}{1 \text{ min}} = \frac{2400 \text{ yards}}{x}$ $100 \cdot x = 1 \cdot 2400$ $\frac{100x}{100} = \frac{2400}{100}$ $x =$
   $x =$ 24 min.

Student B is able to solve the problem by using division, avoiding all the manipulations involved in the proportion algorithm.

Student B

Dan and Alan take turns cutting the grass.
Their lawn is 60 yards long and 40 yards wide.

1. **What is the area of the yard?** A=bh A=60(40) 2,400 **square yards** ✓

Dan takes an hour to cut the lawn using an old mower.

2. **How many square yards does Dan cut in a minute?**
   Show your work.
   $\frac{2400}{60}=40$ ✓
   40 square yards ✓

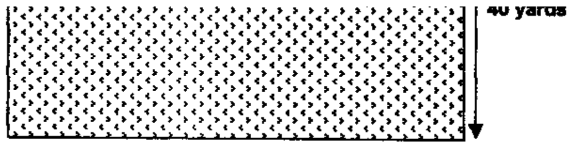
Alan only takes 40 minutes using a new mower.

3. **How many square yards does Alan cut in a minute?**
   Show your calculation.
   $\frac{2400}{40}=60$ ✓
   60 square yards ✓

4. **One day they both cut the grass together.**
   How long do they take?
   Show how you figured it out. 40+60=100sy a
   let x = how long they took min
   $\frac{2400}{100}=x$
   $24=x$ ✓
   24 minutes ✓

For some students, like Student A and B above, this task appears to be a straightforward computational exercise. They recognize a familiar problem-type and use a know tool to solve for the answer. For Student C, this seems to be more of a problem-solving situation. Student C finds a pattern by subtracting the work done by each boy per minute. Student C is then able to reason about the pattern to find an efficient process to calculate the final answer.

Student C



60 yards
× 40 yards
2,400 yards

Dan and Alan take turns cutting the grass.
Their lawn is 60 yards long and 40 yards wide.

1. What is the area of the yard? 2,400 square yards

Dan takes an hour to cut the lawn using an old mower.

2. How many square yards does Dan cut in a minute? 40 square yards
   Show your work.
   $\frac{2,400 \text{ yards} \div 60}{60 \text{ min} \div 60} = \frac{40 \text{ yards}^2}{1 \text{ min}}$

Alan only takes 40 minutes using a new mower.

3. How many square yards does Alan cut in a minute? 60 square yards
   Show your calculation.
   $\frac{2,400 \text{ yards} \div 40}{40 \text{ min} \div 40} = \frac{60 \text{ yards}^2}{1 \text{ min}}$

4. One day they both cut the grass together. 24 minutes
   How long do they take?
   Show how you figured it out.
   2,400 yards = total
   1 min = 2,400 yards − (40 yards + 60 yards) = 2,300 yards · 2
   2 min = 2,300 − 100 = 2,200
   The pattern is that for each minute, you subtract 100 yards that have been cut.
   An easier way to do this is 2,400 yards (total) divided by 100 yards (the number of yards of grass that a being cut every minute) = 24 min to cut all the grass.

8

Student D is another student for whom this task is a problem-solving situation rather than an exercise. The student attempts to use an area model to mark off each section and calculate the time needed per section. Using the approximate scale for the drawing the 5 min sections for Alan would actually take 10 min. So is the student thinking “If each student worked 10 minutes, I could give 5 of those minutes to each person?” or does the student lose track of the dimensions and just assign an arbitrary value to each section? The proportions could have led to a correct solution, but the dimensions for the area calculations are awkward and the final section where one student works 1 min. and the other student works 3 min. seem to indicate the student does not realize that the boys are working simultaneously. What questions could help push the student’s thinking?

Student D

Alan only takes 40 minutes using a new mower.

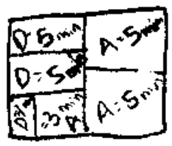
3. **How many square yards does Alan cut in a minute?**
   Show your calculation.

   60 square yards

   $\frac{2400 \text{ (total sq yards)}}{40 \text{ (min he cuts)}} = 60$

4. **One day they both cut the grass together.**
   How long do they take?
   Show how you figured it out.

   24 min.



5 min
5 min
5 min
5 min
1 min
3 min
24 min

Like Student E, many students were able to work successfully on the beginning steps of the problem but did not know how to combine rates in the final part. Student E is able to think about yards per minute for each mower individually, but when combining their rates adds both the 60 and 40 to get 100yds. and adds the minutes to get 2 minutes. This mistake causes the student to think if they each worked 24 minutes then the total time would be 48 minutes.

Student E

Alan only takes 40 minutes using a new mower.

3. **How many square yards does Alan cut in a minute?**
   Show your calculation.

   60 square yards

   $\frac{2400}{40} / \frac{40 \text{min}}{40}$ $\quad 40\overline{)2400}$ = 60

4. **One day they both cut the grass together.**
   How long do they take?
   Show how you figured it out.

   48 min

Alan	60 Square yards/Min
Dan	40 square yards/min

24 48 100 SY/2 min

x2

$100\overline{)2400}$ = 24
-200
400

Student F finds the area each student would mow in a two-minute period. The student then makes a table and extends it until the total area of the lawn is mowed. Unfortunately the student skips 12 minutes when labeling the top of the table.

Student F

Alan only takes 40 minutes using a new mower.

3. How many square yards does Alan cut in a minute?
Show your calculation.

60 square yards a min

$$40\overline{)2400} = 0060$$

4. One day they both cut the grass together.
How long do they take?
Show how you figured it out.

26 minutes

min	2	4	6	8	10	14	16	18	20	22	24	26	
Dan	80	160	240	320	400	480	560						
Alan	120	240	360	480	600	720	840						
	200	400	600	800	1000	1200	1400	1600	1800	2000	2200	2400	

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Many students, about 10%, just added the rates together. If it takes Alan 40 minutes to mow the lawn and Dan 60 minutes, then together it would take 100 minutes to mow the law. Students are doing an operation without thinking about the situation. "Would it take them more time or less time if they worked together?" See the work of Student G.

Student G

4. One day they both cut the grass together.
How long do they take?
Show how you figured it out.

~~1 hr. 40 min.~~

60+40=100 min.
1hr. 40 min.

5

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Other students, like Student H, reason that if one boy can mow the lawn in 40 minutes with two boys the time would be cut in half. The student is not thinking about the different speeds or the effect of the equipment on speeds. Both students cannot use the faster mower simultaneously.

Student H

4. **One day they both cut the grass together. How long do they take? Show how you figured it out.**

$2\overline{)40}$ = 30

20 minutes

2 person's
÷ 40 minuts with one p
20min with two people

Understanding the meaning of working together in a mathematical situation is difficult for students to think about. For many students this means that we should each do half the work, rather than if we start and end at the same time how long will it take us? Student I tries a number of things, but settles for dividing each boys time in half. Student J thinks about dividing the area in half and then finds the amount of time for each boy to mow that area. Student K would have had the same error of 50 min., but makes an error in calculating half the area. Instead of dividing the total area in half, Student K halves the dimensions giving her the area for only 1/4th the original lawn.

Student I

4. **One day they both cut the grass together. How long do they take? Show how you figured it out.**

50 min.

D - 30 min. to cut 1/2
A - 20 min to cut 1/2
50

$2\overline{)2400}$ = 1200

$\frac{1200}{30} = \frac{40}{1}$

Student J

4. **One day they both cut the grass together. How long do they take? Show how you figured it out.**

50 minutes

1200 30
1200 20

Student K

4. One day they both cut the grass together.
How long do they take?
Show how you figured it out.

25 minutes together

The yard slit in half =
A=30x20=600
Dan: 600÷40 = 15 minutes
Alan: 600÷60 = 10 mintues
what needs to be cut
how much he cuts in a minute
15 +10=25 minutes

Some students had difficulty analyzing the effect of operations on dimensions or labels. Students need to have explicit conversations about how dimensions change as a result of multiplication or division. Student L divides yards by yards to get hours.

Student L

Alan only takes 40 minutes using a new mower.

3. How many square yards does Alan cut in a minute?
Show your calculation.

60 yards

2400÷40
60 yards a minute

4. One day they both cut the grass together.
How long do they take?
Show how you figured it out.

2 hours

2400÷1200 = 2 hr.
divid by half so since there it 2 people you cut the work in half so it's 1200.

Often the focus is on what do students need to reach the top, what are the areas of misunderstandings for the students who are meeting standards, but not at the top level. Now, think about what are the holes or gaps for students who can't make a coherent attack on the problem. How are their needs different? The problem is also more complex, because the errors in their logic are not as consistent as the errors made by the "almost there's". Student M is able to calculate area, but has no concept of a rate. The student seems to think about what do I know about yards? There are 3 feet in a yard and I am usually asked to convert from one unit to another. This student needs significant instruction.

Student M

Dan and Alan take turns cutting the grass.
Their lawn is 60 yards long and 40 yards wide.

1. **What is the area of the yard?** 2,400 ✓ **square yards**

Dan takes an hour to cut the lawn using an old mower.

2. **How many square yards does Dan cut in a minute?** 180 sq. yards X
   Show your work. 1 yard = 3 feet

   $60 \times 3 = 180$ $\quad$ $40 \times 3 = 120$ X

Alan only takes 40 minutes using a new mower.

3. **How many square yards does Alan cut in a minute?** 120 sq. yards X
   Show your calculation.

   $40 \times 3 = 120$ X

4. **One day they both cut the grass together.**
   How long do they take? 20 min X
   Show how you figured it out. 40 min $\frac{1}{2}$ is 20 min

   X

Again Student N has learned to calculate area. The logic for parts 2 and 3 are not as clear. The student seems to think the tasks call for finding factors of the amount of time given. Which factors are chosen and which factors are not is undefined by the student. In part 4 Student N makes the common mistake of combining the times for both boys. However the student does not understand time and time conversions, because the student uses 1hr. and 40 min. as if it were equivalent to 140 min. instead of 100 min.

Student N

Dan and Alan take turns cutting the grass.
Their lawn is 60 yards long and 40 yards wide.

60
x40
00
2400
2400

1. What is the area of the yard? 2400 ✓ **square yards**

Dan takes an hour to cut the lawn using an old mower.

2. How many square yards does Dan cut in a minute? 2 Square X
Show your work. Dan cuts with an old lawn mower, 2 because 30·2=60 and he has the old lawn mowero X

Alan only takes 40 minutes using a new mower.

3. How many square yards does Alan cut in a minute? 4 Squares X
Show your calculation.
Well I did 4 ÷ 40 = and I got 10 and 10 x 4 = 40 so the answer is X

4. One day they both cut the grass together.
How long do they take? 70 minutes X
Show how you figured it out.
1 hour and 40 minutes
1:40 minutes divided by 2 is 70 so it took them 1 hour and 10 minutes X

Student O struggles with the idea of how operations effect units or dimensions. The student divides hours by minutes to get square yards and minutes by minutes to get square yards. All of these last three students need work around understanding the concept of use of ratio in everyday contexts before receiving more procedural instruction.

Student O

Dan and Alan take turns cutting the grass.
Their lawn is 60 yards long and 40 yards wide.

1. What is the area of the yard? 60 yards × 40 yards = 2400 — **2400** square yards ✓

Dan takes an hour to cut the lawn using an old mower.

2. How many square yards does Dan cut in a minute? — **1 sq yard** X
   Show your work.
   60 hour ÷ 60 min = 1 sq yard X

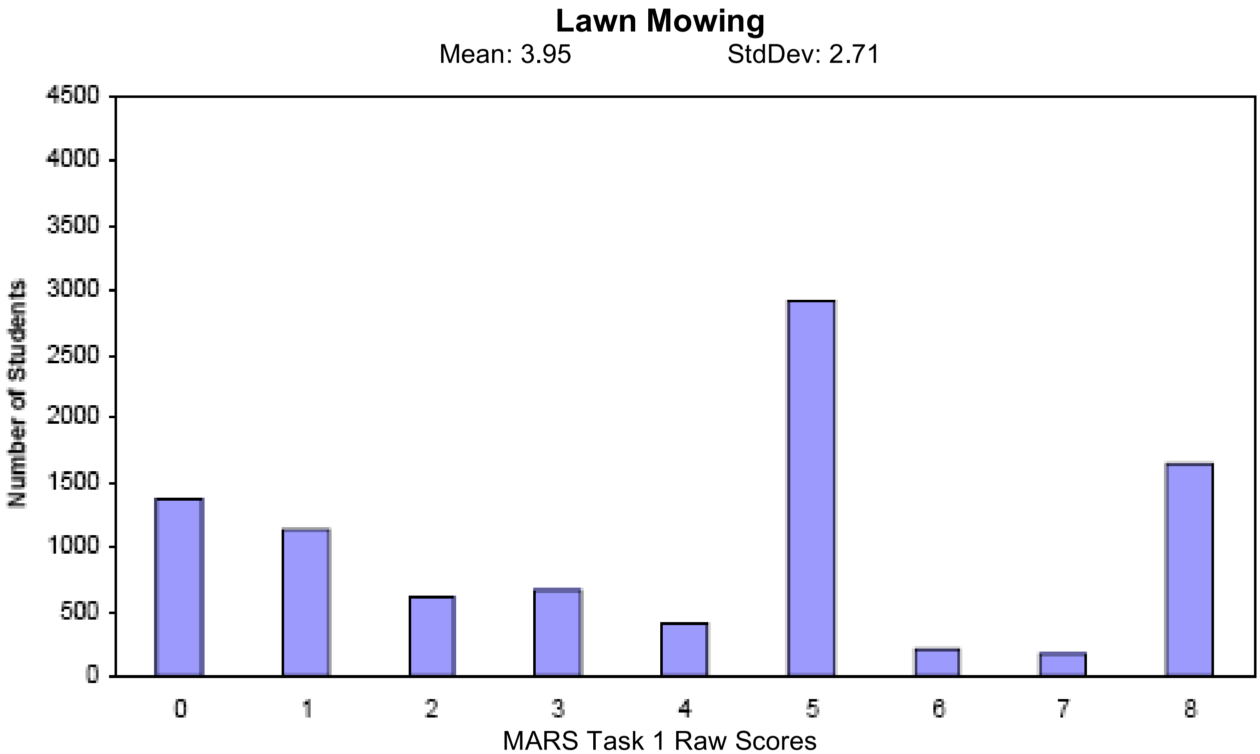
Alan only takes 40 minutes using a new mower.

3. How many square yards does Alan cut in a minute? — **1.5 sq yards** X
   Show your calculation.
   60 min ÷ 40 min = 1.5 yards X

4. One day they both cut the grass together. — **30 min** X
   How long do they take?
   Show how you figured it out.
   60 min ÷ 2 people = 30 min X

Teacher Notes:

Frequency Distribution for Task 1 – Grade 7 – Lawn Mowing



Score:	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Student Count	1382	1141	614	664	412	2914	204	173	1647
% < =	15.1%	27.6%	34.3%	41.5%	46.0%	77.9%	80.1%	82.0%	100.0%
% > =	100.0%	84.9%	72.4%	65.7%	58.5%	54.0%	22.1%	19.9%	18.0%

The maximum score available on this task is 8 points.
The minimum score for a level 3 response, meeting standards, is 4 points.

Most students, about 85%, could calculate the area of the lawn. Many the students, 65%, could find the area and calculate the rates for each boy to mow the lawn or show a correct process for calculating the rate. More than half the students, 58%, could find the area, calculate one rate correctly showing the work and either calculate or show the process for finding the other rate. Half the students could find the area and calculate both rates and show the process for the calculation. 18% of the students could meet all the demands of the task, including reasoning about combining rates, understanding that both boys worked simultaneously, and find the solution for total time to mow the lawn together. 15% of the students scored no points on this task. All of the students in the sample attempted the problem.

Lawn Mowing

Points	Understandings	Misunderstandings
0	All the students in the sample attempted the problem.	Almost 10% of the students found perimeter instead of area in part 1 of the task. About 5% could not multiply with the zeros and got 240 instead of 2400 sq. yds. Another 5% just added the two sides together to get 100 yds. *(Some students may have gotten points in other parts of the task, which is why this adds to more than the 13% who scored zero.)*
1	Students could calculate area.	They did not know how to reason about finding rates. They may have tried to convert minutes to yards to feet. The most common rates for part 2 are 60, 4, and 1. The most common rates for part 3 are 6, 20, and 2.5.
3	Students could calculate the area and usually find both rates or they could show a correct process for finding rates.	Students struggled with showing their work or carrying out the correct computations.
4	Students could calculate the area and usually find both rates or they could show one correct process for finding rates.	Students either did not document one of the first three tasks or made a calculation error.
5	Students could calculate area and rates for mowing the lawn.	Students could not reason about combining rates. Many students thought about each person cutting their time in half or sharing the area equally, rather than thinking about both boys working simultaneously (13%). More than 10% of the students just added the times for both together. Almost 10% of the students did not attempt this part of the task.
8	Students could calculate area and rates and reason about how to combine those rates when both boys worked together.	

Based on teacher observations, this is what seventh graders knew and were able to do:

- Calculate area of a field
- Find the rate for mowing a lawn in yards per minute

Areas of difficulty for seventh graders:

- Understanding the idea of rates
- Combining rates
- Thinking about people working simultaneously rather than doing half the area
- Mathematical notation to document their thinking (*many students put things like 60/2400, when they meant 2400 divided by 60)*

Strategies used by successful students:

- Setting up ratios or proportions
- Making tables and expanding them to find the time for both boys
- Use labels to reason about the quantities being calculated

Questions for Reflection on Lawn Mowing:

- How are your students introduced to the idea of rates? Do they get ample opportunities to discuss common rates like miles per hour, miles per gallon, words per minute, copies per minute, money per hour, heartbeats per minute, etc.?
- Look at your standard teacher's edition that you currently use for planning instruction. Does your textbook organize the mathematical ideas using rate as a "big mathematical idea" or is it interspersed throughout the book as individual, isolated facts? This is important to think about.
- How might you design a unit to help students think about and make sense of rates and their applications?
- Do you think students could find their own strategies for solving the problems? What types of strategies might students come up with? What kinds of questions would help connect all the strategies? What kind of generalizations would you want students to make about rate problems?
- When working with rate problems, students need to have a clear understanding of units and be able to use unit or dimensional analysis. What opportunities do students in your class have to discuss the effects of operations on units? What are the norms for your classroom on using labels? How do labels add to the sense-making of the context? Make a table listing the types of labels your students used in the "show your work" section of the task.

Look at student work for part one. How many of your students could compute:

Area 2,400	Perimeter 200	Made calculation error – 240	Added the two sides – 100	Other

Look at student work in part two. How many students had answers of:

40	4	60	20	2	1	Other

- What might the student have been thinking to get these answers? How do different errors point to different experiences that students need to overcome these misconceptions?
- Many students had difficulty thinking about how to combine rates in part 4. Did your students:
  - Understand that the students were working simultaneously and use that to combine rates to correctly solve part 4? List the different strategies students used that led them or could have led them to the correct solution. Do students in your class have access or feel comfortable with a variety of strategies?
  - Think about halving the amount of time for each child?
  - Think about halving the area between to two children?
  - Cut either Alan's or Dan's time in half?

Implications for Instruction:

In looking at the thinking of young children in work around subtraction, we know that problems with the same numbers seem like different problems to the student. Even though as adults we would use subtraction for the problems, students will model different actions for the problems, like taking away, adding on, or comparing the size of two sets. For students starting to make sense of any new idea, the context and problem type seem so different that the student doesn't notice that the numbers are the same. While for adults, the first three parts of the problem may seem like variations of a number fact family, the problems are different enough to make them a problem- solving task for many students. Teachers need to provide a variety of problem-types for working with rates, so that students will start to recognize them as a category of problems. Students are not fluent with the idea until they can work the problems forwards and backwards as an exercise, rather than as a new "problem-solving" event.

Rates are a huge mathematical idea. Teachers need to develop a comprehensive, cohesive unit to develop understanding of different types of rates, their applications in the real world, strategies for solving rate problems, and their representations in proportions and as slopes. Textbooks may not be the best resource for planning such a unit. One series I looked at devoted a whole chapter to understanding rates as a big mathematical idea. Students were given the opportunity to see the relationship between several types of rates and how methods for solving them were all the same. The second series I examined had rates scattered throughout the book, almost as an afterthought. Each type of rate was given a separate formula, specific to that example. Students were asked to solve maybe two problems with that particular rate. No opportunities were given for the student to make sense of the idea of a rate or try to make sense for solving the problem on their own. Rate was just an opportunity to practice substituting values into a formula. Teachers may want to work together in study groups to develop or organize the materials in a more unified approach.

An important idea for working with rates is the idea of dimensional analysis. Students should be given opportunities to discuss how labels change because of operations of multiplication and division. Students should also see and discuss examples of using dimensional analysis to find errors in reasoning. For example, does it make sense to divide hours by hours and get yards? Why? Why not? Can you multiply money times money? Why? Why not?

Successful students should have a variety of strategies to work with rates, such as setting up proportions, finding unit rates, using double number lines, making tables. Students should be asked questions to help them generalize across the strategies. How are all the strategies the same? What is different about the strategies? Which strategy is best used in this particular situation? Why do you think that? Which strategy is most efficient? Why might certain strategies be more prone to errors? Thinking about the general situation helps students develop mathematical power. It is time consuming and cumbersome to solve each new problem as a brand new category. Students need to start making connections between problems and problem types.

Students need help learning to think about combining rates. Students need to understand that the two events are occurring simultaneously, both things occur until the whole task is completed. This is different from a fair- sharing situation.

Teacher Notes:

Performance Assessment Task

Lawn Mowing
Grade 7

This task challenges a student to use rates and proportional reasoning to solve problems. A student must be able to understand conversions between standard measures, such as area and time, to calculate rates and to reason about combining rates to accomplish a chore. A student must be able to construct a rationale for how two rates work together to complete a chore.

Common Core State Standards Math - Content Standards

<u>Ratios and Proportional Relationships</u>

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. *For example, if a person walks ½ mile in each ¼ hour, compute the unit rate as the complex fraction ½ / ¼ miles per hour, equivalently 2 miles per hour.*

7.RP.2 Recognize and represent proportional relationships between quantities.

c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationships between the total cost and the number of items can be expressed as t=pn.*

7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

<u>Geometry</u>

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Common Core State Standards Math – Standards of Mathematical Practice

MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP.6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of

the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.				
Grade Level	Year	Total Points	Core Points	% At Standard
7	2005	8	4	58%