Rugs
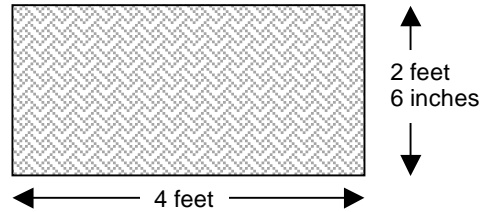
This problem gives you the chance to:
• find perimeters of shapes
• use Pythagoras' Rule

Hank works at a factory that makes rugs.

The edge of each rug is bound with braid. Hank's job is to cut the correct length of braid for each rug.

1. The factory makes a rectangular rug that is 4 feet long and 2 feet 6 inches wide.
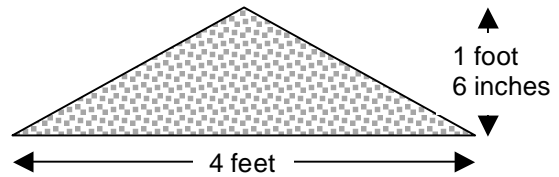
    How much braid will Hank need to cut to go all the way around this rug?

    _____ feet

    Show your work.

2. The factory makes a triangular rug. It is an isosceles triangle 4 feet wide with a perpendicular height of 1 foot 6 inches.

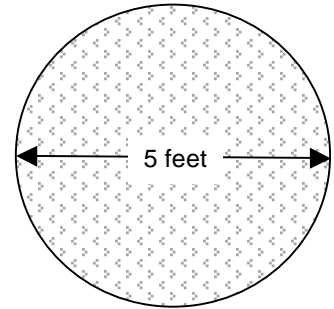   How much braid will Hank need to cut to go all the way around this rug?

    _____ feet

    Show your work.

Rugs   Test 8

3. The factory also makes a circular rug that has a diameter of 5 feet.

How much braid will Hank need to go all the way around
this circular rug? Give your answer in whole feet.

> *The circumference of a circle = 2πr*
> *The area of a circle = πr²*

_____feet

Show your work.

4. There are plans to make a semi-circular rug which also has a diameter of 5 feet. Hank thinks that
this rug will need half as much braid as the circular rug.

Explain why Hank is not correct.

_____

_____

_____

_____

How much braid will this rug need?   _____feet

8

Task 2: Rugs		Rubric
The core elements of performance required by this task are: • find perimeters of shapes • use Pythagoras' Rule Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives a correct answer: **13** feet **and** shows correct work such as: 2 x (4 + 2.5)	1	1
2. Gives a correct answer: **9** feet Shows correct work such as: Attempts to use the Pythagorean Rule. $x^2 = 2^2 + 1.5^2 = 6.25$ $x = 2.5$ 2.5 + 2.5 + 4 Addition of sides.	1 1 1ft	3
3. Gives a correct answer: **16** feet or 5π feet Shows correct work such as: 5 x	1 1	2
4. Gives a correct explanation such as: The curved part would be half the length of the circumference of the circle but you would need to add on 5 feet for the straight edge. Gives correct answer: 13 feet	1 1	2
Total Points		**8**

Rugs

Work the task. Look at the rubric. What are the big mathematical ideas this task is assessing?

_____

_____

_____

Look at student work for part one of the task. How many of your students put:

13	48	12	1.4	6.5	13 ft. 2 in.	Other

Which students were confusing perimeter and area?
How were they making sense of the 2 feet 6 inches when they switched to decimals?
What other types of errors did you find?

Now look at work of finding the perimeter of a triangle. How many of your students put:

9	7	24	11	18	4	6	Other

How many of your students confused height of triangle with side length?
How many tried to apply an area formula?
Did they understand that 6 inches is half a foot?
What other misconceptions led to the errors above?
How many students attempted to use Pythagorean Theorem in some way?

Now look at work on circumference of a circle. How many of your students put:

16 or 5π	15.7	No answer	10	31.4	Other

Can you figure out how students got the 10 or 31.4? What were their misconceptions?
Did you see evidence of students using the area formula instead of the circumference
formula? Did they try to use both formulas?

In part 4, circumference for half the circular rug, how many of your students put:

13	No response	5	15	15.7	8	2.5	10	Other

Can you think why students made some of these errors? What types of misconceptions
do these errors represent?

Why do you think this task was so difficult for students? What are the implications for
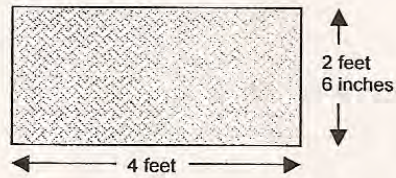instruction?

Looking at Student Work on Rugs

Student A is able to think about 6 in. representing half a foot and convert fluently between decimal and measurement notation. The student understands that to find the length of the sides for the triangle he must apply Pythagorean theorem. In part 3 the student shows the calculations and describes the rounding process. Notice the diagram to help clarify the situation in part 4.

Student A

Hank works at a factory that makes rugs.

The edge of each rug is bound with braid. Hank's job is to cut the correct length of braid for each rug.

1. The factory makes a rectangular rug that is 4 feet long and 2 feet 6 inches wide.

2 feet 6 inches

4 feet

How much braid will Hank need to cut to go all the way around this rug?

_____13_____ feet ✓

Show your work.

2 ft 6 in
4 ft 6 in
= 4 ft 14 ft = 13 ft ✓

2. The factory makes a triangular rug. It is an isosceles triangle 4 feet wide with a perpendicular height of 1 foot 6 inches.
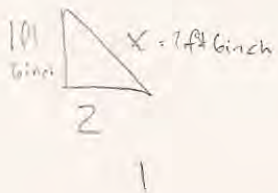
1 foot 6 inches

4 feet
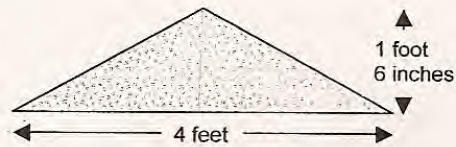
How much braid will Hank need to cut to go all the way around this rug?

_____9_____ feet ✓

Show your work.

1 ft 6 inch = 1,5 ft

1ft 6inc   x = 2ft 6inch
2

1

$1,5^2 + 2^2 = x^2$ x = 6.25 ✓

2 ft 6inch
2 ft 6inch
5 ft
4 ft
9 ft ✓

$\sqrt{6.25} = 2.5$ ✓
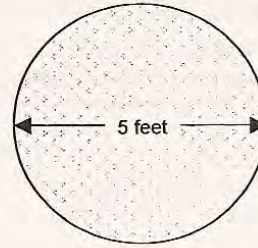
Page 2   Rugs   Test 8

Student A, part 2

The factory also makes a circular rug that has a diameter of 5 feet.

How much braid will Hank need to go all the way around this circular rug? Give your answer in whole feet.

The circumference of a circle = $2\pi r$
The area of a circle = πr^2

_____16_____ feet ✓

Show your work.

$5 \div 2 = 2.5 = r$

$2 \cdot \pi \cdot 2.5 \approx 15.70$ ✓

15.7 rounded off to _16_.

There are plans to make a semi-circular rug which also has a diameter of 5 feet. Hank thinks that this rug will need half as much braid as the circular rug.

Explain why Hank is not correct.

Hank is incorrect because it's not a circle anymore and half of the circular rug wouldn't cover the 5ft across the bottom.

How much braid will this rug need? _____13_____ feet ✓

$15.7 \div 2 = 7.85 + 5 = 12.85$

12.85 rounded off to 13 ft.
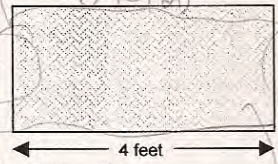
5ft

12.85 ft.

8 Ⓑ

Page 3

Rugs Test 8

Student B shows dividing up the triangular rug to make 2 right triangles. Notice the "?" marks on the diagram to indicate what needs to be calculated. In part 3 the student forgets to round to the nearest whole feet. The explanation in part four is good when cutting the rug there is a new side that is not accounted for and then the diagram shows where the new side is and how long it is.

Student B

Hank works at a factory that makes rugs.

The edge of each rug is bound with braid. Hank's job is to cut the correct length of braid for each rug.

1. The factory makes a rectangular rug that is 4 feet long and 2 feet 6 inches wide.

How much braid will Hank need to cut to go all the way around this rug?
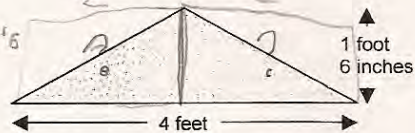
Show your work.

$$2.5 \times 2 = 5.0$$

$$4 \times 2$$

13 feet

2. The factory makes a triangular rug. It is an isosceles triangle 4 feet wide with a perpendicular height of 1 foot 6 inches.

How much braid will Hank need to cut to go all the way around this rug?

Show your work.

$$1.5^2 + 2^2 = c^2 \times 2$$

$$2.25 + 4 = \sqrt{6.25}$$

$$4 \text{ feet} + 2.5 + 2.5$$
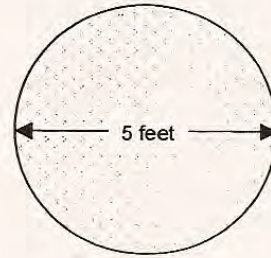
$$6.5 + 2.5$$

9 feet

Page 2

Rugs Test 8

Student B, part 2

3. The factory also makes a circular rug that has a diameter of 5 feet.

How much braid will Hank need to go all the way around this circular rug? Give your answer in whole feet.

> The circumference of a circle = $2\pi r$
> The area of a circle = πr^2

15.7 ✗ feet

Show your work.

diameter is r·2   So 5 ÷ 2 = r = 2.5

Pi = 3.14

2 = 2

2·3.14·2.5 = 15.7 feet need ✓

4. There are plans to make a semi-circular rug which also has a diameter of 5 feet. Hank thinks that this rug will need half as much braid as the circular rug.

Explain why Hank is not correct.

Because altogh a Semi circle has half area it has anew Side that is not accounted for. ✓

How much braid will this rug need? 12.85 ✓ feet

$\dfrac{15.7}{2}$

7.85
+5.00
12.85

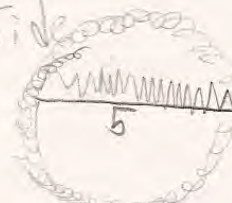$-\dfrac{15.7}{2}$ because half the circular curve

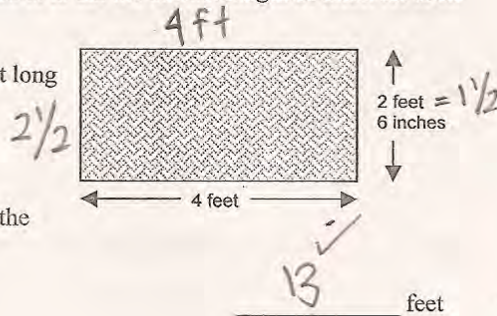+5 cause diameter = to that side

5

8

Student C understands how to use Pythagorean theorem to find the length of the sides of the triangular rug, but loses sight of what is being asked by the problem, "How much braid to go all the way around the rug?" *How do we help students to define what is being asked? Do students get enough opportunities to work on tasks where they use one calculation to help find something else? What types of word problems do students encounter in your classroom on a regular basis?* In part 3 the student forgets to round. *Do students have enough experiences talking about significant digits and using rounding in context?* With the increased use of technology, students need more opportunities to think what level of accuracy makes sense for this purpose or this situation.

Student C

Hank works at a factory that makes rugs.

The edge of each rug is bound with braid. Hank's job is to cut the correct length of braid for each rug.

1. The factory makes a rectangular rug that is 4 feet long and 2 feet 6 inches wide.
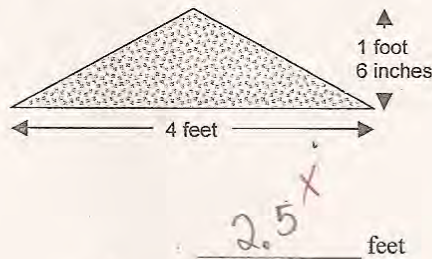
   4 ft

   2 feet 6 inches = 1½

   2½

   4 feet

   How much braid will Hank need to cut to go all the way around this rug?

   Show your work.

   $2.5 + 2.5 + 4 + 4 = 13$ ✓

   13 _____ feet

2. The factory makes a triangular rug. It is an isosceles triangle 4 feet wide with a perpendicular height of 1 foot 6 inches.

   1 foot 6 inches

   4 feet

   How much braid will Hank need to cut to go all the way around this rug?

   Show your work.

   2.5 _____ feet

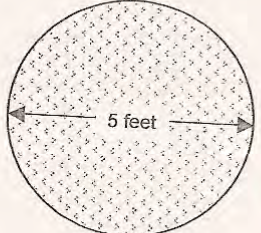 $C = \sqrt{2^2 + 1.5^2} = \sqrt{4 + 2.25} = \sqrt{6.25} = 2.5$

   1.5

   2

Student C, part 2

3. The factory also makes a circular rug that has a diameter of 5 feet.

How much braid will Hank need to go all the way around this circular rug? Give your answer in whole feet.

> The circumference of a circle = $2\pi r$
> The area of a circle = πr^2

Show your work.

_15.7_ feet

5 feet

$C = \pi d$

$(3.14)(5)$

15.7

4. There are plans to make a semi-circular rug which also has a diameter of 5 feet. Hank thinks that this rug will need half as much braid as the circular rug.

Explain why Hank is not correct.

Its because Hank forgot to divide 15.7 by 2, then get the answer he recieved, 7.85, to add by 5 which you will get 12.85 as your answer.

How much braid will this rug need?   _12.85_ feet

5ft

$\dfrac{15.7}{2} = 7.85$

$\begin{array}{r} 7.85 \\ +\ 5 \\ \hline 12.85 \end{array}$
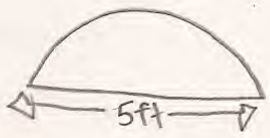
Students had a difficult time with this task. So it is important to analyze where their thinking breaks down. Student D understands that the situation in part 2 calls for use of the Pythagorean theorem. However the student does not take the square root of 6.25 or c^2. The student also forgets to add the length for the base of the rug in part 2. In part 3 the student chooses the correct formula for circumference, changes the diameter to a radius in order to use the formula, and calculates correctly. However, the student does not realize that the circumference is for the whole circle. The student makes a diagram to understand part 4 which could have helped her solve the problem, but doesn't know how to use the information in the diagram. *What question might you want to ask this student?*

Student D

Hank works at a factory that makes rugs.

The edge of each rug is bound with braid. Hank's job is to cut the correct length of braid for each rug.

1. The factory makes a rectangular rug that is 4 feet long and 2 feet 6 inches wide.
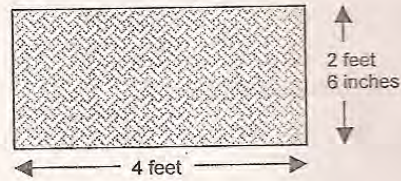
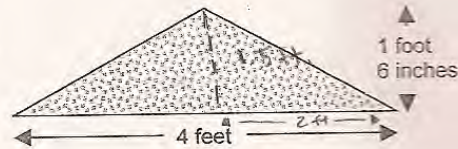   How much braid will Hank need to cut to go all the way around this rug?

   _____ 13 ✓ feet

   Show your work. ✓
   $P = (4+4) + (2.5 + 2.5)$
   $P = 8 + 5$
   $P = 13$

2. The factory makes a triangular rug. It is an isosceles triangle 4 feet wide with a perpendicular height of 1 foot 6 inches.

   How much braid will Hank need to cut to go all the way around this rug?

   _____ 12.50 ✗ feet

   Show your work.
   Pythagora's Rule :

   $c^2 = 1.5^2 + 2^2$ ✓
   $c^2 = 2.25 + 4$
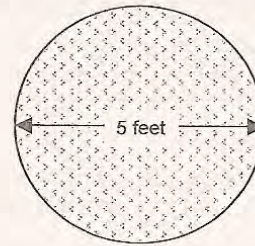 $c^2 = 6.25$
   $c = 6.25$

   $P = 6.25 + 6.25 = 12.50$

Student D, part 2

3. The factory also makes a circular rug that has a diameter of 5 feet.

   How much braid will Hank need to go all the way around this circular rug? Give your answer in whole feet.

   | The circumference of a circle = $2\pi r$ |
   | The area of a circle = πr^2 |

   _31.4_ ✗ feet

   5 feet

   𝒪

   Show your work.

   $C = 2\pi r$

   $C = 2 \cdot 3.14 \, (5 \div 2)$

   $c = 2 \cdot 3.14 \cdot 2.5$

   $c = 6.28 \cdot 2.5$

   $C = 15.70$ is half ✗ circle so 15.70 ✗ $\cdot 2 = 31.4$

   𝒪

4. There are plans to make a semi-circular rug which also has a diameter of 5 feet. Hank thinks that this rug will need half as much braid as the circular rug.
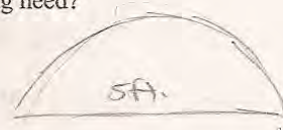
   Explain why Hank is not correct. ✗

   𝒪

   _____

   _____

   _____

   _____

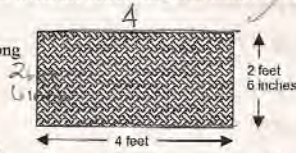   How much braid will this rug need? _____ ✗ feet   𝒪

   5ft.

Student E confuses the height for the side length in part 2. In part 3 the student uses diameter instead of radius in the formula for circumference. The student understands the situation in part 4 and solves the problem correctly, using the incorrect information from part 3.

Student E

Hank works at a factory that makes rugs.

The edge of each rug is bound with braid. Hank's job is to cut the correct length of braid for each rug.

1. The factory makes a rectangular rug that is 4 feet long and 2 feet 6 inches wide.

   How much braid will Hank need to cut to go all the way around this rug?

   Show your work.

   _13_ ✓ feet 1 1

2. The factory makes a triangular rug. It is an isosceles triangle 4 feet wide with a perpendicular height of 1 foot 6 inches.

   How much braid will Hank need to cut to go all the way around this rug?
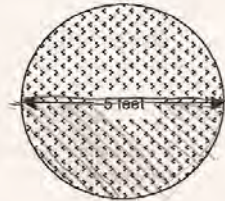
   Show your work.

   _11_ X feet 0 0

3. The factory also makes a circular rug that has a diameter of 5 feet.

   How much braid will Hank need to go all the way around this circular rug? Give your answer in whole feet.

   | The circumference of a circle = $2\pi r$ |
   | The area of a circle = πr^2 |

   _31_ X feet

   Show your work.

   $2\pi 5 = 31.4159265A$

   round = 31

4. There are plans to make a semi-circular rug which also has a diameter of 5 feet. Hank thinks that this rug will need half as much braid as the circular rug.

   Explain why Hank is not correct.

   Hank is not correct because when he's doing the flat Part of the semi-circle he's going to need less than half. ✓

   How much braid will this rug need?   _20.5_ X feet

   5 comes from the flat part

 $31 \div 2 = 15.5$ ✓

   $15.5 + 5 = 20.5$ ✓

Student F tries to simplify the calculations in part 2 by correctly converting feet to inches. However, instead of using Pythagorean theorem to find the length of the sides, the student finds the area of the smaller triangle and then divides by 2 instead of multiplying by 2 to find the total area. The student can choose the correct formula in part 3 and do the calculations in part 3. Even though the student can make a diagram for part 4, the student does not "see" the added side made by the cut. *What opportunities do students in your class have to make sense of diagrams?*

Student F

Hank works at a factory that makes rugs.

The edge of each rug is bound with braid. Hank's job is to cut the correct length of braid for each rug.

1. The factory makes a rectangular rug that is 4 feet long and 2 feet 6 inches wide.

How much braid will Hank need to cut to go all the way around this rug?

   2 feet 6 inches
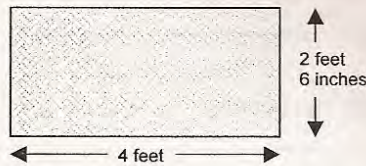
   4 feet

   13 ✓ feet I
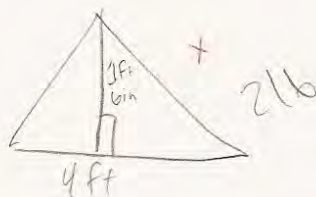
Show your work.

2 ft 6 in
2 ft 6 in
4 ft 12 in = 5 ft + 8 ft = 13 ft

2. The factory makes a triangular rug. It is an isosceles triangle 4 feet wide with a perpendicular height of 1 foot 6 inches.

   1 foot 6 inches

   4 feet

How much braid will Hank need to cut to go all the way around this rug?

   18 ✗ feet 0 .

Show your work.

   0 .
   0
   0 .

4 ft
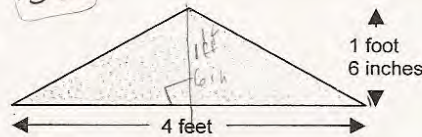
2ft × 1 ft 6in
48 in × 18 in =
864 ÷ 2 =
432 = 36 ÷ 2 = 18
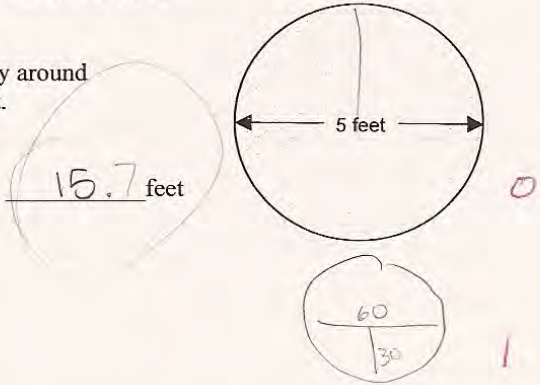
Student F, part 2

3. The factory also makes a circular rug that has a diameter of 5 feet.

How much braid will Hank need to go all the way around this circular rug? Give your answer in whole feet.

X

_____15.7_____ feet

5 feet

O

> The circumference of a circle = $2\pi r$
> The area of a circle = πr^2

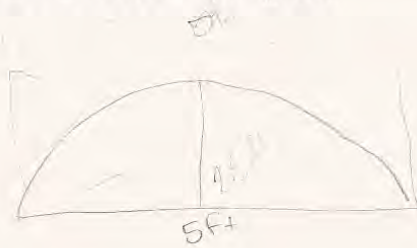Show your work.

$2 \times 3.14 \times 30 =$

$188.4 \text{ in} = 15.7 \text{ ft}$

60
30

1

4. There are plans to make a semi-circular rug which also has a diameter of 5 feet. Hank thinks that this rug will need half as much braid as the circular rug.

Explain why Hank is not correct.

i don't know why; i would also take half as much X

O.

O

How much braid will this rug need?   X 7.85 X feet

5ft

2 (2)

Student G uses distributive property to make sense of multiplication on measurements and ease the conversion between inches and feet. However the student mistakes the height for the side length in part 2. In part 3 the student uses the diameter instead of the radius to calculate the circumference. The student also makes place value errors in multiplying decimals. The student seems to know that there will be a new side of 5 feet in part 4, but doesn't add it to the previous answer. Maybe the size of answer in part 3 obscured the final step.

Student G

Hank works at a factory that makes rugs.

The edge of each rug is bound with braid. Hank's job is to cut the correct length of braid for each rug.

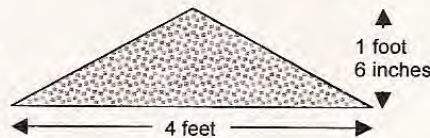1. The factory makes a rectangular rug that is 4 feet long and 2 feet 6 inches wide.
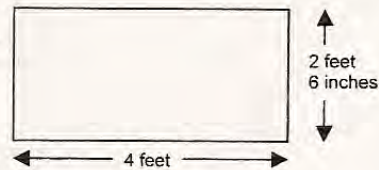
   How much braid will Hank need to cut to go all the way around this rug?

   _____13_____ ✓ feet   1

   Show your work.

   $$4^{ft} \quad 2^{ft} \quad 6^{in}$$
   $$\times 2 \quad \times 2 \quad \times 2$$
   $$8^{ft} + 4^{ft} + 12^{in} \text{ or } 1 \text{ foot} = 13 \text{ feet}$$

2. The factory makes a triangular rug. It is an isosceles triangle 4 feet wide with a perpendicular height of 1 foot 6 inches.

   How much braid will Hank need to cut to go all the way around this rug?

   _____7_____ ✗ feet   0

   Show your work.

   $$1^{ft} \quad 6^{in} \quad \times$$
   $$\times 2 \quad \times 2 \quad \times$$
   $$4^{ft} + 2^{ft} + 12^{in} \text{ or } 1 \text{ foot} = 7^{ft}$$
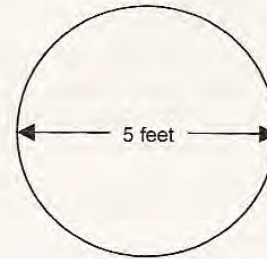
   0

Student G, part 2

3. The factory also makes a circular rug that has a diameter of 5 feet.

How much braid will Hank need to go all the way around this circular rug? Give your answer in whole feet.

The circumference of a circle = 2πr
The area of a circle = πr²

3440 X feet ?

Show your work.

```
  3.14
+ 3.14
   4
  6.28
× 5.
  0008
3440.
```
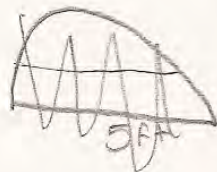
4. There are plans to make a semi-circular rug which also has a diameter of 5 feet. Hank thinks that this rug will need half as much braid as the circular rug.

Explain why Hank is not correct.

Hank is not correct Because you need barid for more sides on the rug. X

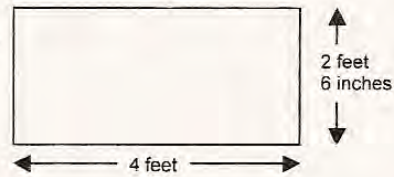How much braid will this rug need? _____ X feet

Z 8

Student H does not understand that 6 inches is half of a foot. Notice the decimal notation. In part 1 the student tries to use an area formula instead of calculating the perimeter. In part 2 the student tries to apply an area formula for triangle, but forgets to divide by 2. In part 3 the student uses the diameter instead of the radius in the formula and incorrectly chooses the area formula instead of the circumference formula. The student does understand the situation in part 4 and should have received follow through points for the solution.

Student H

Hank works at a factory that makes rugs.

The edge of each rug is bound with braid. Hank's job is to cut the correct length of braid for each rug.

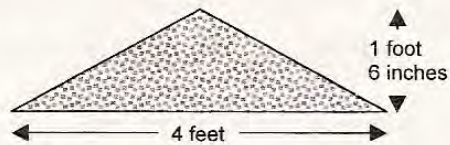1. The factory makes a rectangular rug that is 4 feet long and 2 feet 6 inches wide.

   How much braid will Hank need to cut to go all the way around this rug?

   Show your work.

   $$\begin{array}{r} 4 \\ \times\ 2.6 \\ \hline 10.4 \end{array}$$

   _____10.4_____ feet

2. The factory makes a triangular rug. It is an isosceles triangle 4 feet wide with a perpendicular height of 1 foot 6 inches.

   How much braid will Hank need to cut to go all the way around this rug?

   Show your work.

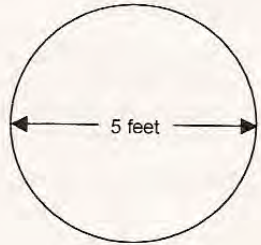 $$\begin{array}{r} 4 \\ \times\ 1.6 \\ \hline 6.4 \end{array}$$

   _____6.4_____ feet

Student H, part 2

3. The factory also makes a circular rug that has a diameter of 5 feet.

How much braid will Hank need to go all the way around this circular rug? Give your answer in whole feet.

| The circumference of a circle = $2\pi r$ |
| The area of a circle = πr^2 |

_____ 16 ~~X~~ feet

5 feet

$\mathscr{D}$

Show your work.

2·π·5

3.14·5²

3.14

$\begin{array}{r} 25 \\ \times\ 3.14 \\ \hline 15.70 \\ 62.50 \\ \hline 74.50 \end{array}$

$\begin{array}{r} 78.54 ~X \\ -\ 30 \\ \hline 48.54 \end{array}$

$\mathscr{O}$

4. There are plans to make a semi-circular rug which also has a diameter of 5 feet. Hank thinks that this rug will need half as much braid as the circular rug.

Explain why Hank is not correct.

Hank is not correct because for the rug to need half as much it wouldn't be closed. ⌒ ✓

How much braid will this rug need?   29 ~X~ feet $\mathscr{D}$

$\begin{array}{r} 24 \\ 2\overline{)48} \\ -4 \\ \hline 08 \\ -8 \\ \hline 0 \end{array}$
$\begin{array}{r} 24 \\ +\ 5 \\ \hline 29 \end{array}$

8th Grade   Task 2   Rugs

Student Task	Find perimeters of shapes. Use Pythagorean theorem to find side lengths.
Core Idea 4 Geometry and Measurement	**Apply appropriate techniques, tools and formulas to determine measurements.** • Create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship.

Based on teacher observations, this is what eighth graders know and are able to do:
- Find the perimeter of a rectangle
- Calculate the circumference
- Convert 6 inches to decimal notation

Areas of difficulty for eighth graders:
- Using Pythagorean theorem to find a side length
- Rounding numbers in context
- Confusing diameter and radius
- Confusing area and perimeter
- Making sense of a diagram, recognizing the added edge when cutting a circle in half
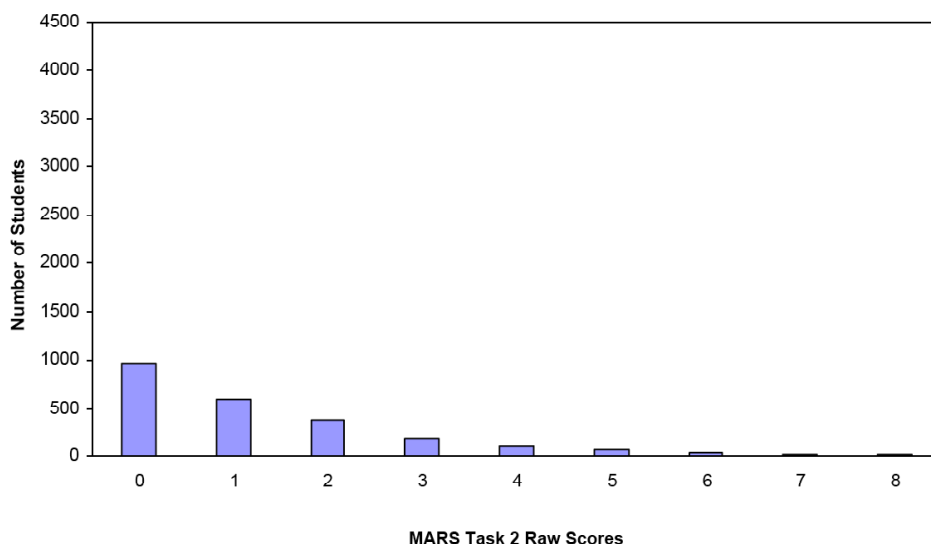
Task 2 - Rugs

Mean: 1.31   StdDev: 1.58

Table 41: Frequency Distribution of MARS Test Task 2, Grade 8

Task 2 Scores	Student Count	% at or below	% at or above
0	970	40.9%	100.0%
1	601	66.2%	59.1%
2	375	82.0%	33.8%
3	191	90.1%	18.0%
4	103	94.4%	9.9%
5	72	97.4%	5.6%
6	35	98.9%	2.6%
7	13	99.5%	1.1%
8	13	100.0%	0.5%

Figure 50: Bar Graph of MARS Test Task 2 Raw Scores, Grade 8

MARS Task 2 Raw Scores

The maximum score available on this task is 8 points.
The minimum score for a level 3 response, meeting standards, is 3 points.

Many students, about 60%, could find the perimeter of a rectangle. Only 18% of the students could meet standard on this task usually getting the area of the rectangle and finding the circumference of a circle rounded to the nearest whole number. About 10% could also explain why the perimeter of a semi-circle is not half the circumference of a circle. Less than 1% of the students could meet all the demands of the task including using Pythagorean theorem to find the side length of an isosceles triangle, find the perimeter of a semicircle, and round the circumference of a circle to the nearest whole number. Almost 41% of the students scored no points on this task. 91% of the students with this score attempted the task.

Rugs

Points	Understandings	Misunderstandings
0	91% of the students with this score attempted the task.	Students did not know how to find the perimeter of a rectangle. Almost 6% confused area and perimeter. The students who calculated area used 2.6 ft for 2 ft. 6 in. An additional 5% converted 2 ft. 6 in. to 2.6 ft when calculating perimeter. About 5% just added two dimensions when calculating area. Almost 7% ignored the 6 inches and just add 4+4+2+2 to 12 inches for the perimeter.
1	Students could find the perimeter of a rectangle.	Students could not use the formula to find the circumference of a circle. 7.5% of the students did not attempt this part of the task. 12% used the diameter instead of the radius for calculating the circumference. 15% had an answer of 10. 18% did not round 15.7 to 16 ft.
3	Students could find the perimeter of a rectangle and calculate and round the circumference of circle when given the diameter.	Students could not reason about finding the perimeter of a semicircle. Almost 22% of the students did not attempt this part of the task. 9% of the students thought the answer would be 10. Almost 7% thought the perimeter would be exactly half the circumference of the circle. Almost 7% thought the perimeter would equal the diameter, ignoring the curved part of the rug. Other popular answers were 15, 2.5, and 8.
4		Students could not find the perimeter of an isosceles triangle, given the base and height. 7% did not attempt this part of the task. 16% of the students used the height as the side length to get an answer of 7ft. 15% of the students multiplied base times height (area without the divide by 2). 2/3 of this group used 1.6 ft. instead of 1/5 ft. Other popular answers were 18 and 4.
8	Students could use Pythagorean theorem to find the side length of an isosceles triangle and calculate perimeter of a rectangle and triangle. Most could convert between units of measurement and decimal notation. Students could find the circumference of a circle given the diameter and find the perimeter of a semicircle.	

Implications for Instruction

Students need more experiences with geometric figures. At this grade level students should be able to compose and decompose shapes into parts. For example, they should see that if you cut a circular rug in half, the new perimeter will be larger than half the circumference of a circle because the perimeter will also include the line across the center of the circle or the diameter. Students should then be able to calculate the perimeter of a semi-circle.

Students at this grade level should have experience working with Pythagorean Theorem and applying it in problem situations. An interesting task for interpreting diagrams is the 2001 Course 2 task, Writing Desks.

Many students still struggled with the idea of area and perimeter. Some students confused radius and diameter. Students need practice applying these ideas to problems to help them sort through or make sense of the differences.

Ideas for Action Research

Because 41% of the students scored no points on this task and most of them attempted the problem. It is important to think about what are the mathematical gaps in their thinking. What types of experiences do these students need?

Often when planning remediation or helping students who are behind, teachers think about the students who are almost there. What are the few steps they need to be successful? But what is it that the students who are at the lowest end of the spectrum need? How are their issues different?

Sit down with colleagues and examine the following pieces of student work or examples from your own students. Consider the following questions:

1. What are the strengths, if any, that the student has? What are the concepts the students understand about the situation? How might these strengths be used to help build their understanding of the whole situation?
2. How did students use representations? Were the representations accurate? Why or why not? What would have helped the student to improve their representation?
3. What misunderstandings does the student have? What skills is the student missing? What does this suggest about a specific course of action to help this student?
4. How are the needs of each of these students the same or different?

After you have carefully looked at each piece of student work, see if you can devise a plan of experiences/ discussions/ tools that might help these students to make more sense of these situations. While you don't have these exact students in your class, each member of the group will probably have students with similar misunderstandings. Identify students who you think are low and plan different approaches for attacking the problems outlined here. Have each person in the group try out a different course of action and report back on the how the lesson or series of lessons effected the targeted students. See if you can all use some similar starting problems and bring work of the students to share. What types of activities or experiences made the most noticeable improvement in student work?

Anna

Hank works at a factory that makes rugs.

The edge of each rug is bound with braid. Hank's job is to cut the correct length of braid for each rug.

1. The factory makes a rectangular rug that is 4 feet long and 2 feet 6 inches wide.

   2 feet 6 inches
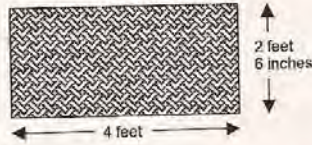
   4 feet

   How much braid will Hank need to cut to go all the way around this rug?

   12

   _____ 8 _____ feet

   Show your work.

   6
   4
   12
   12

2. The factory makes a triangular rug. It is an isosceles triangle 4 feet wide with a perpendicular height of 1 foot 6 inches.

   1 foot 6 inches

   4 feet

   How much braid will Hank need to cut to go all the way around this rug?

   _____ 4 _____ feet

   Show your work.

   6
   + 1
   7

3. The factory also makes a circular rug that has a diameter of 5 feet.

   How much braid will Hank need to go all the way around this circular rug? Give your answer in whole feet.

   5 feet

   The circumference of a circle = $2\pi r$
   The area of a circle = πr^2

   _____ 10 _____ feet

   Show your work.

   5
   × 2
   10
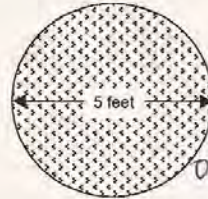
4. There are plans to make a semi-circular rug which also has a diameter of 5 feet. Hank thinks that this rug will need half as much braid as the circular rug.

   Explain why Hank is not correct.

   I think he is correct

   _____

   _____

   _____
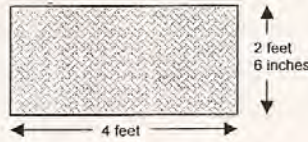
   How much braid will this rug need?   _____ 3 _____ feet

Bradley

The edge of each rug is bound with braid. Hank's job is to cut the correct length of braid for each rug.

1. The factory makes a rectangular rug that is 4 feet long and 2 feet 6 inches wide.

   2 feet 6 inches

   4 feet

   How much braid will Hank need to cut to go all the way around this rug?

   10.5 ✗ feet

   Show your work.

   $6+4=10$

   $2 \frac{5}{8} \frac{5}{18}$

   ✗

2. The factory makes a triangular rug. It is an isosceles triangle 4 feet wide with a perpendicular height of 1 foot 6 inches.

   1 foot 6 inches

   4 feet

   How much braid will Hank need to cut to go all the way around this rug?

   5.1 ✗ feet

   Show your work.

   $1+4=5$

   $5 \overline{)6} \frac{1}{-5} \frac{}{1}$
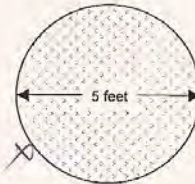
   ✗

3. The factory also makes a circular rug that has a diameter of 5 feet.

   5 feet

   How much braid will Hank need to go all the way around this circular rug? Give your answer in whole feet.

   The circumference of a circle = $2\pi r$
   The area of a circle = πr^2

   ✗ 15.70 feet ✗

   Show your work.

   $A \square \pi r^2 = (3.14)(5) = 15.70$ ✓ ✓

4. There are plans to make a semi-circular rug which also has a diameter of 5 feet. Hank thinks that this rug will need half as much braid as the circular rug.

   Explain why Hank is not correct.

   Hank is wrong because they are diffrent mensuramants.
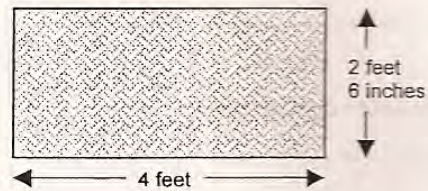
   How much braid will this rug need? 15.70 ✗ feet ()

Carrie

Hank works at a factory that makes rugs.

The edge of each rug is bound with braid. Hank's job is to cut the correct length of braid for each rug.

1. The factory makes a rectangular rug that is 4 feet long and 2 feet 6 inches wide.

    2 feet 6 inches

    4 feet

    How much braid will Hank need to cut to go all the way around this rug?

    Show your work.

    $4 + 4 = 8$ $12/12$

    $2/6 + 2/6 = 4/12$

    12×0 feet

2. The factory makes a triangular rug. It is an isosceles triangle 4 feet wide with a perpendicular height of 1 foot 6 inches.

    1 foot 6 inches

    4 feet
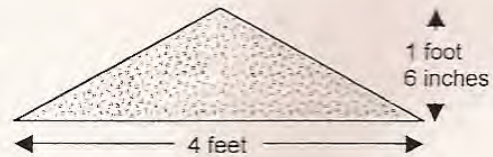
    How much braid will Hank need to cut to go all the way around this rug?

    Show your work.

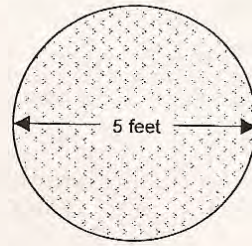 6×0 feet

    $4 + 1 + 1 = 6$

    0

    0

Carrie, part 2

3. The factory also makes a circular rug that has a diameter of 5 feet.

How much braid will Hank need to go all the way around this circular rug? Give your answer in whole feet.

| The circumference of a circle = $2\pi r$ |
| The area of a circle = πr^2 |

16.7 feet

Show your work.

3.14
x   2
_____
6.28

✓

2 × 3.14 × 2.5   6.28
x   2.5
_____
3 1 4 0   16,700
1 3 5 6 0

4. There are plans to make a semi-circular rug which also has a diameter of 5 feet. Hank thinks that this rug will need half as much braid as the circular rug.

Explain why Hank is not correct.

I think that you would only need half not half more.

How much braid will this rug need?   2.5 feet

3.14
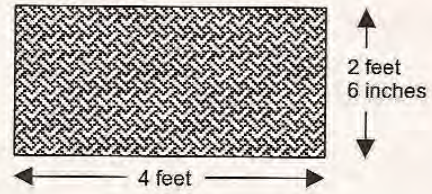x 2.5
_____
15 7 0 0
< 2 8 0
_____
7.09 8 0

5 ÷ 2 = 2.5

8

David

Hank works at a factory that makes rugs.

The edge of each rug is bound with braid. Hank's job is to cut the correct length of braid for each rug.

1. The factory makes a rectangular rug that is 4 feet long and 2 feet 6 inches wide.

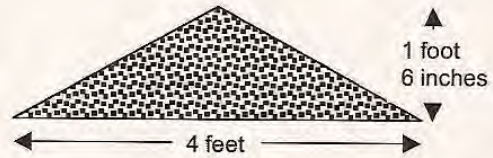   How much braid will Hank need to cut to go all the way around this rug?

   ___21___ feet

   Show your work.

   4×4 = 16 + 2.5×2.5 = 21 he needs 21ft

   2ft 6in basically equals ~2.5 on a ruler

2. The factory makes a triangular rug. It is an isosceles triangle 4 feet wide with a perpendicular height of 1 foot 6 inches.

   How much braid will Hank need to cut to go all the way around this rug?

   ___19___ feet

   Show your work.

   4×4 = 16 +

   1 + 1 = 2ft + 6in + 6in = 1ft = 3

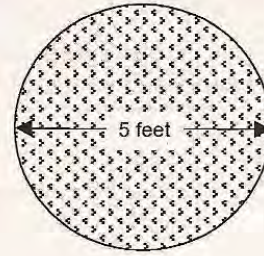# David, part 2

3. The factory also makes a circular rug that has a diameter of 5 feet.

How much braid will Hank need to go all the way around this circular rug? Give your answer in whole feet.

| The circumference of a circle $= 2\pi r$ |
| The area of a circle $= \pi r^2$ |

__15.7__ feet

5 feet

Show your work.

$5 \div 2 = 2.5 = radius$

$2.5 \times 3.14 = 7.85 \times 2 = 15.7 feet$ ✓

4. There are plans to make a semi-circular rug which also has a diameter of 5 feet. Hank thinks that this rug will need half as much braid as the circular rug.

Explain why Hank is not correct.

He is not correct because you can't make a semi-circular rug with the amound of material that you made circular rug with.
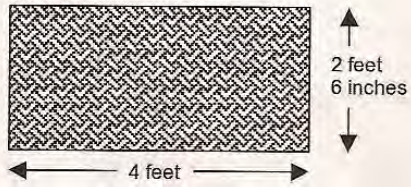
How much braid will this rug need?   around 8 feet
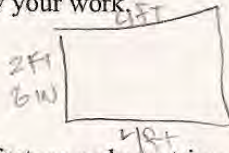
Ellie

The edge of each rug is bound with braid. Hank's job is to cut the correct length of braid for each rug.

1. The factory makes a rectangular rug that is 4 feet long and 2 feet 6 inches wide.

   How much braid will Hank need to cut to go all the way around this rug?

   _____13____ feet
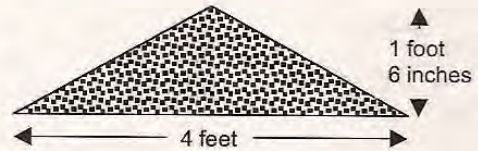
   Show your work.

   $$4 \quad 2.6$$
   $$+4 \quad +2.6$$
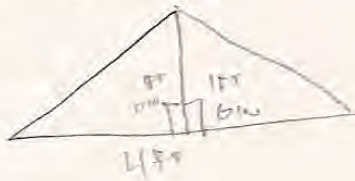 $$8 \quad 5.2 = 13FT.$$

2. The factory makes a triangular rug. It is an isosceles triangle 4 feet wide with a perpendicular height of 1 foot 6 inches.

   How much braid will Hank need to cut to go all the way around this rug?

   _____7_____ feet

   Show your work.

   $$4 + 1.76$$
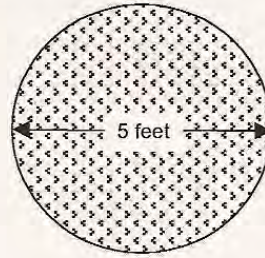 $$+ 1.76$$
   $$2.12$$
   $$= 3$$
   $$4 + 3 = 7$$

Ellie, part 2

e factory also makes a circular rug that has a diameter of 5 feet.

How much braid will Hank need to go all the way around
this circular rug? Give your answer in whole feet.

The circumference of a circle = $2\pi r$
The area of a circle = πr^2

19.63 ✗ feet

5 feet

Show your work.

$$2.5 \times 3.14 = 7.85 \times 2 = 15.7$$

$$6.25 \times 3.14 = 19.62 \approx 19.63$$

4. There are plans to make a semi-circular rug which also has a diameter of 5 feet. Hank thinks that this rug will need half as much braid as the circular rug.

Explain why Hank is not correct.

BECAUSE SINCE THE CIRCLE IS CUT
IN HALF THE 5FT CANT BE THE RADIUS
BECAUSE ITS ALREADY CUT IN HALF

How much braid will this rug need? _54.95 ✗ feet_

$$31.4$$
$$78.5$$
$$109.9$$

$$2 \overline{)109.9} = 54.95$$
$$10$$
$$09$$
$$8$$
$$19$$
$$18$$
$$10$$

8

Page 3

Rugs Test 8

Fred

Hank works at a factory that makes rugs.

The edge of each rug is bound with braid. Hank's job is to cut the correct length of braid for each rug.

1. The factory makes a rectangular rug that is 4 feet long and 2 feet 6 inches wide.

2 feet
6 inches

4 feet

$4 + 4 + 2 + 2 = 12 + 12$
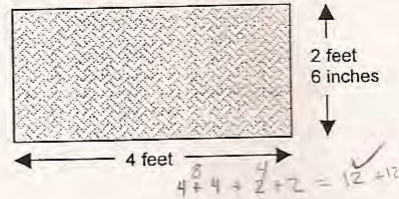
How much braid will Hank need to cut to go all the way around this rug?

Show your work.

_____13_____ feet

2. The factory makes a triangular rug. It is an isosceles triangle 4 feet wide with a perpendicular height of 1 foot 6 inches.

1 foot
6 inches

4 feet

How much braid will Hank need to cut to go all the way around this rug?

Show your work.

$8 + 1.6 + 1.6 ✗$

$3 + 8 = ✗$
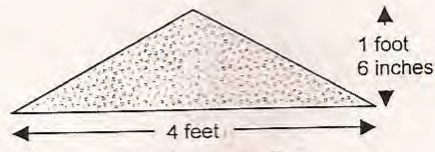
_____11 ✗_____ feet

Fred, part 2

3. The factory also makes a circular rug that has a diameter of 5 feet.

How much braid will Hank need to go all the way around this circular rug? Give your answer in whole feet.

The circumference of a circle = $2\pi r$
The area of a circle = πr^2
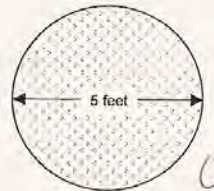
_____2.5 ✗_____ feet

5 feet

Show your work.

$3.14 × 5^2 =$

3.14
$× 77$
1570
3280
4850 ✗

4. There are plans to make a semi-circular rug which also has a diameter of 5 feet. Hank thinks that this rug will need half as much braid as the circular rug.

Explain why Hank is not correct.

_____

Geena

The edge of each rug is bound with braid. Hank's job is to cut the correct length of braid for each rug.

1. The factory makes a rectangular rug that is 4 feet long and 2 feet 6 inches wide.

2 feet 6 inches
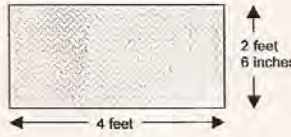
4 feet

How much braid will Hank need to cut to go all the way around this rug?   8 ft
6 inches x

Show your work.

x _____ feet

2. The factory makes a triangular rug. It is an isosceles triangle 4 feet wide with a perpendicular height of 1 foot 6 inches.

1 foot 6 inches

4 feet
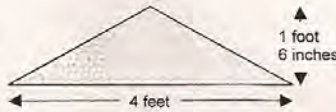
How much braid will Hank need to cut to go all the way around this rug?   4 ft
6 inches

Show your work.

x _____ feet

3. The factory also makes a circular rug that has a diameter of 5 feet.

5 feet

How much braid will Hank need to go all the way around this circular rug? Give your answer in whole feet.

The circumference of a circle = 2πr
The area of a circle = πr²

x
742480 feet

Show your work.

3.14   4985.96
          5
3.14 × 472480
1256
3140
4985.96

4. There are plans to make a semi-circular rug which also has a diameter of 5 feet. Hank thinks that this rug will need half as much braid as the circular rug.
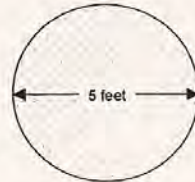
Explain why Hank is not correct.

because he might need more rug because just because the rug is smaller doesn't mean it needs less braid   x

How much braid will this rug need?   256400.00 feet

4985.96
2.50
1 00000
4024800
4971200
56400.00

Performance Assessment Task
Rugs Grade 8
The task challenges a student to demonstrate understanding of the concepts of irrational numbers, the Pythagorean Theorem and use it to calculate perimeter/circumference. A student must understand how to calculate the perimeter/circumference of different figures including rectangles, triangles and circles. A student must be able to make use of common irrational numbers such as the square root of 2 and pi. A student must approximate irrational numbers with a rational number to determine an approximate perimeter of a geometric shape. A student must apply the Pythagorean Theorem to calculate the side of a triangle.
Common Core State Standards Math - Content Standards
The Number System **Know that there are numbers that are not rational, and approximate them by rational numbers.** 8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). **Geometry** **Understand and apply the Pythagorean Theorem**. 8.G.7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
Common Core State Standards Math – Standards of Mathematical Practice
MP.2 Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. **MP.5 Use appropriate tools strategically.** Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.
Assessment Results
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core

points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
8	2007	8	3	18%