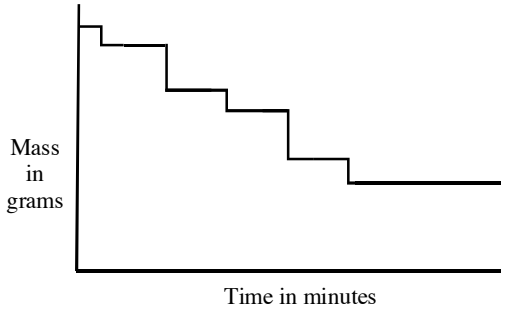
Vincent's Graphs

This problem gives you the chance to:
- interpret graphs
- draw a graph

Vincent is eating a packet of raisins.

This graph shows the changes in the mass of raisins in the packet as time passes.



1.a. What is Vincent doing when there is a vertical line on the graph?

_____

b. Why are the vertical lines of different lengths?

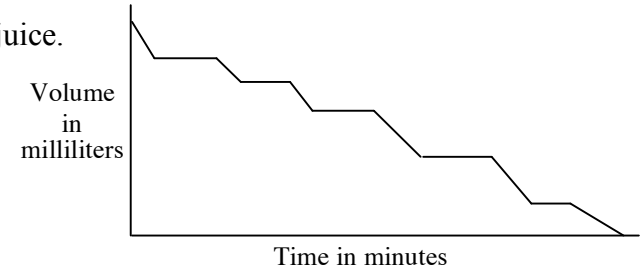
_____

c. Did Vincent eat all the raisins? _____
Explain how you know.

_____

2. Ellie is drinking with a straw from a box of fruit juice.

The graph shows the volume of juice in the box as time passes.



a. What is happening when the line on the graph is horizontal?

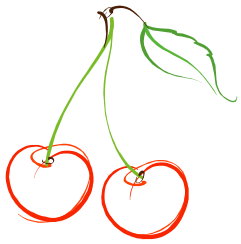
_____

b. Why do the lines going downwards on this graph go at an angle?

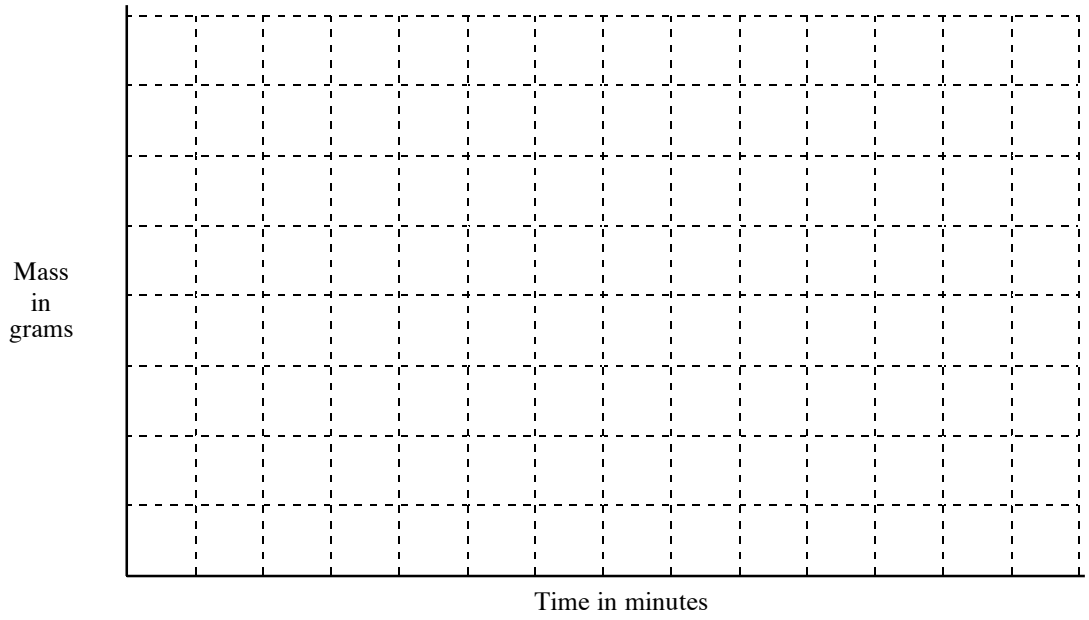
_____

3. Ralph is eating cherries from a bag.

After eating a cherry he puts the stone back into the bag before taking out the next cherry.



On the grid draw a graph to show the changes in the mass of the bag of cherries as time passes.



8

Vincent's Graphs	Rubric	
The core elements of performance required by this task are: • interpreting graphs • drawing graphs Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. a. Gives correct answer such as: he is taking raisins out of the packet.	1	
b. Gives correct answer such as: he takes different numbers of raisins from the packet.	1	
c. Gives correct answer: **No** **and** a correct explanation such as: the line does not reach the x axis.	1	
		3
2. a. Gives correct answer such as: Ellie is not drinking.	1	
b. Shows correct work such as: the volume decreases steadily as the juice is sucked out.	1	
		2
3. Draws a correct graph: First a short horizontal line	1	
Followed by a short line downwards.	1	
A short horizontal line followed by a short line upwards.	1	
The line upwards should be shorter than the first line downwards.		3
Total Points		**8**

Vincent's Graph

Work the task and look at the rubric. What are the key mathematical ideas being assessed in the task?

_____

Look at student work on part 1. How many of your students:

- Showed an understanding that taking raisins out of the box is the cause of the change in weight?_____
- Thought the vertical line was because of eating the raisins?_____
- Said Vincent had stopped eating?_____
- Made comments about the labels rather than referring to the context?_____
- Thought the vertical line was about time?_____
- Couldn't attempt an explanation?_____

Look at student work on part 2. How many students thought the horizontal line was:

- Ellie not drinking?_____
- Ellie drinking?_____
- The speed of Ellie drinking?_____
- Something moving up and down or high and low?_____
- Couldn't attempt an explanation?_____

Now look at student graphs. How many of your students:

- Drew a correct graph?_____
- Drew a graph showing a series of horizontal, vertical lines making a downward staircase?
- Drew graphs with an increasing height?_____
- Drew graphs going up and down, but making a horizontal pattern never going lower on the height?

What other things did you notice about student graphs?

Why do you think students had so much difficulty relating the graph in context?
Do students need more experiences with scale and measuring?
What kinds of experiences and discussions have students had with time graphs?
How do you help them connect the action of the story to shape of the graph? Do they have exposure to graphs where the action of the story is the reverse of the graph?

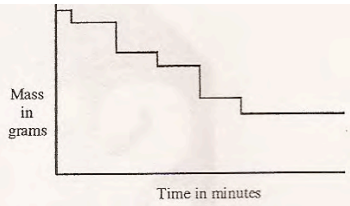
Looking at Student Work on Vincent's Graphs

For the most part Student A is able to correlate parts of the graph to the context of the story. The student doesn't understand that removing the raisins from the box is what changes the weight. The student is able to make some progress on the graph, but does not return the cherry pits to the bag.

Student A

Vincent is eating a packet of raisins.

This graph shows the changes in the mass of raisins in the packet as time passes.



1.a. What is Vincent doing when there is a vertical line on the graph?

He is chewing. ✗ 0

b. Why are the vertical lines of different lengths?

He could be grabbing handfulls of raisins. ✓ 1

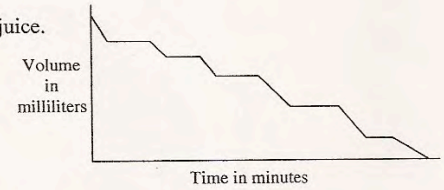
c. Did Vincent eat all the raisins? No ✓

Explain how you know.

The line didn't reach the bottom of the graph. ✓

2. Ellie is drinking with a straw from a box of fruit juice.

The graph shows the volume of juice in the box as time passes.

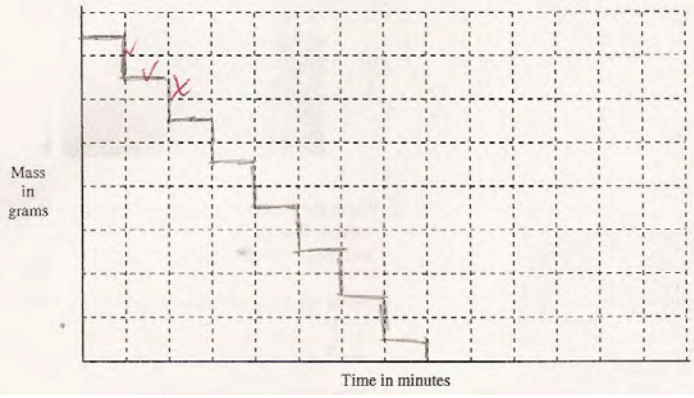


a. What is happening when the line on the graph is horizontal?

She is taking a momentary pause from drinking. ✓

b. Why do the lines going downwards on this graph go at an angle?

She is taking long sips. ✓



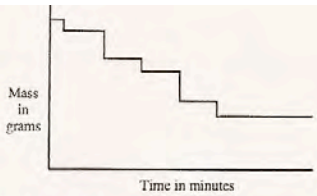
1
0

Student B makes the common mistake of taking the action of the graph as eating rather than removing raisins from the box. While the actions are connected, it is the removing the raisins that causes the instantaneous change in weight. Student B is one of the few students able to make a correct graph in part 3.

Student B

Vincent is eating a packet of raisins.

This graph shows the changes in the mass of raisins in the packet as time passes.



1.a. What is Vincent doing when there is a vertical line on the graph? 0

Vincent is eating the raisins and there are less and less in the box

b. Why are the vertical lines of different lengths?

There are different lengths because he eats the raisins at different speeds 0

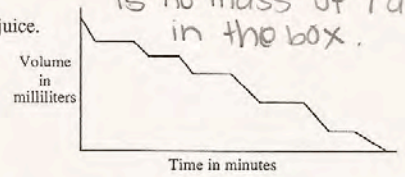
c. Did Vincent eat all the raisins? NO

Explain how you know.

IF he were done the line would've gone straight down meaning there is no mass of raisins in the box. 1

2. Ellie is drinking with a straw from a box of fruit juice.

The graph shows the volume of juice in the box as time passes.



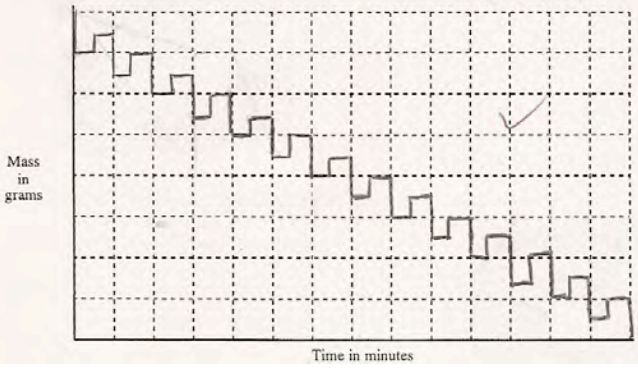
a. What is happening when the line on the graph is horizontal? 1

ellie takes a break of drinking juice

b. Why do the lines going downwards on this graph go at an angle? 0

Its because how long ellie takes to drink juice.

On the grid draw a graph to show the changes in the mass of the bag of cherries as time passes.



1
1
1

Student C also confuses eating in part 1a and 1b with the action of removing raisins. The student has good explanations for the other parts. The student doesn't understand the instantaneous action of removing from and adding objects to a scale and therefore has slanted versus vertical lines on the graph. The student allows no time for eating the cherry between removal and return of the pit.

Student C

Time in minutes

1.a. What is Vincent doing when there is a vertical line on the graph?

Eating the raisons when vertical line.

b. Why are the vertical lines of different lengths?

Vincent was eating a different amount of raisons.

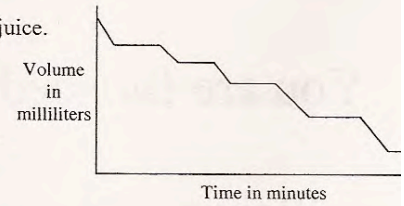
c. Did Vincent eat all the raisins? no

Explain how you know.

because the line doesn't drop to the bottom of the graph

2. Ellie is drinking with a straw from a box of fruit juice.

The graph shows the volume of juice in the box as time passes.



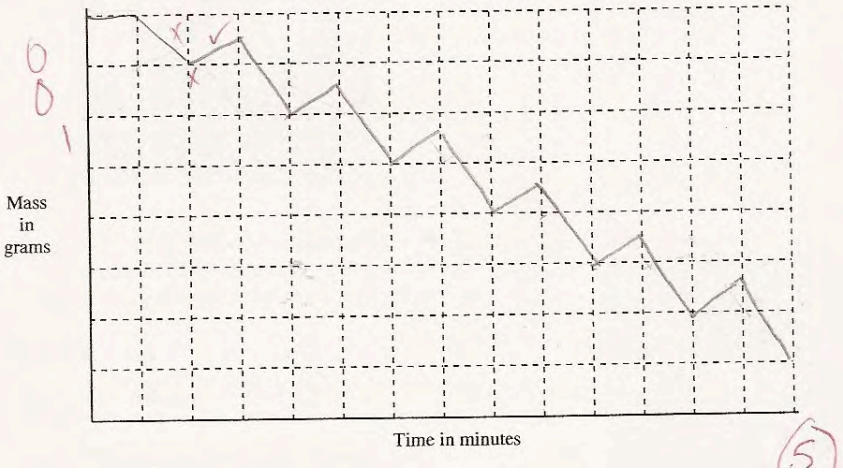
a. What is happening when the line on the graph is horizontal?

Ellie stops drinking the juice.

b. Why do the lines going downwards on this graph go at an angle?

because liquid isn't ~~ever~~ sipped up in even amounts. ~~so~~

On the grid draw a graph to show the changes in the mass of the bag of cherries as time passes.

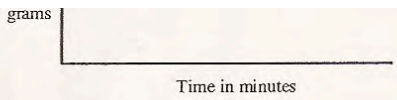


(5)

Student D is starting to focus on the scale of the graphs rather than trying to match the action of the story to the context of the graph. In part 1c the student does not give a full explanation. *What does it mean for the line to stop for a while?* In part 2 the student makes the common mistake of the thinking the vertical line is drinking or time to drink. Again both answers in 2 are more about scales (in this case the wrong scales) than the context. *How do we help students to connect the shape of graph to the real life action being represented? What does the shape have to do with the story?*

Student D

mass of raisins in the packet as time passes.



1.a. What is Vincent doing when there is a vertical line on the graph? ✓ 1

He started to eat smaller ammounts of raisins. The raisins are decreasing.

b. Why are the vertical lines of different lengths?

It is how much raisins Vincent ate during the minutes. X 0

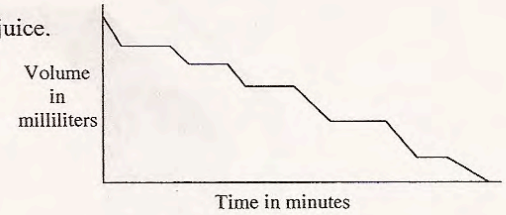
c. Did Vincent eat all the raisins? yes

Explain how you know.

The line in the graph has stopped for a while. X 0

2. Ellie is drinking with a straw from a box of fruit juice.

The graph shows the volume of juice in the box as time passes.



a. What is happening when the line on the graph is horizontal?

How long she is drinking the juice. X 0

b. Why do the lines going downwards on this graph go at an angle?

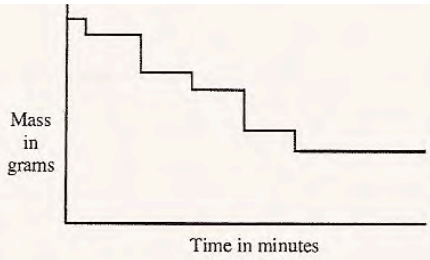
It is how much juice is left and how long it took her to drink everything. X 0

Look at the work of Student E. *What do you think the student really understands about the situations? What questions would you like to ask the student?*

Student E

Vincent is eating a packet of raisins.

This graph shows the changes in the mass of raisins in the packet as time passes.



1.a. What is Vincent doing when there is a vertical line on the graph?

The vertical line means the raisin is a different size ×0

b. Why are the vertical lines of different lengths? 0

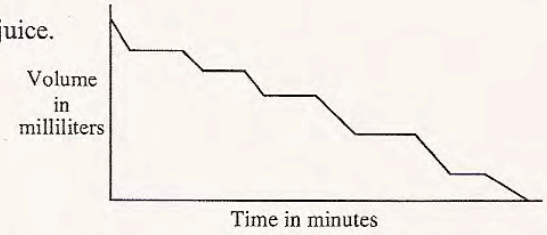
c. Did Vincent eat all the raisins? NO ✓ 1

Explain how you know.

The line on the graph doesn't reach the bottom.?

2. Ellie is drinking with a straw from a box of fruit juice.

The graph shows the volume of juice in the box as time passes.



a. What is happening when the line on the graph is horizontal? 1

He is not drinking the juice. ✓

b. Why do the lines going downwards on this graph go at an angle?

They go at an angle because he could be drinking different amounts. ✗

Student F is an example of a student with very little ability to make sense of the graph. The student thinks about the context and what actions could or could not be happening, but does not relate them to scale or slope. *How do we help students make these connections?*

Student F

Time in minutes

1.a. What is Vincent doing when there is a vertical line on the graph?

He's not eating raisins. ✗ 0

b. Why are the vertical lines of different lengths? 0

✓

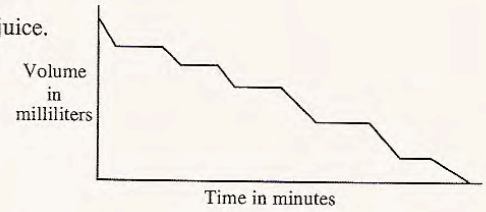
c. Did Vincent eat all the raisins? yes ✗ 0

Explain how you know.

Because he stoped eating them which means no more.

2. Ellie is drinking with a straw from a box of fruit juice.

The graph shows the volume of juice in the box as time passes.



a. What is happening when the line on the graph is horizontal? 1

he's not drinking juice. ✓

b. Why do the lines going downwards on this graph go at an angle? 0

because he drinks and ✗ ~~stopes~~ Stops

Student G is focused on the labels of the graph. *How is this different from relating the graph to the context?*

Student G

Time in minutes

1.a. What is Vincent doing when there is a vertical line on the graph?

Vincent is eating the raisins ... x C

b. Why are the vertical lines of different lengths?

the lines represent the mass of the raisins left in the box for every minute x C

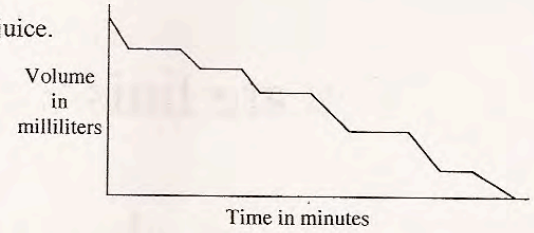
c. Did Vincent eat all the raisins? yes

Explain how you know.

the last vertical line keeps going x C

2. Ellie is drinking with a straw from a box of fruit juice.

The graph shows the volume of juice in the box as time passes.



a. What is happening when the line on the graph is horizontal?

she is drinking x C

b. Why do the lines going downwards on this graph go at an angle?

time passed by as she is drinking x C

Student H appears to be reading the graph from bottom to top, rather than top to bottom. The student talks about the mass growing, mass gained. The student does not notice the downward trend of the graph. Student I shows an example of students reversing the slope of the graph.

Student H

Time in minutes

1.a. What is Vincent doing when there is a vertical line on the graph?

He is graphing the amount of mass that is grown. X 0

b. Why are the vertical lines of different lengths?

It shows how much mass the packet gained ✓ 0

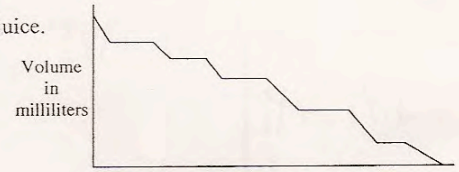
c. Did Vincent eat all the raisins? He didn't X

Explain how you know.

As the mass get higher it means he ate them, but it didn't get to the top. 0

2. Ellie is drinking with a straw from a box of fruit juice.

The graph shows the volume of juice in the box as time passes.



a. What is happening when the line on the graph is horizontal?

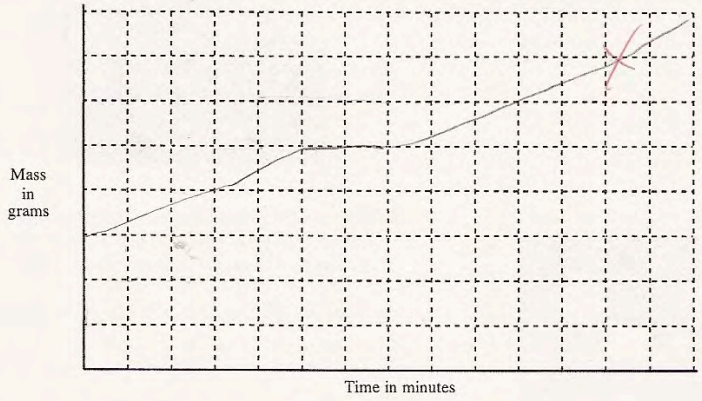
It means time is passing by. X 0

b. Why do the lines going downwards on this graph go at an angle?

Because he is drinking a liquid. ✓ 0

Student I

On the grid draw a graph to show the changes in the mass of the bag of cherries as time passes.



Student J is not thinking of the labels or the context of the action. The student only notices the up or downward trend of the graphs. *What experiences does this student need?*

Student J

Time in minutes

1.a. What is Vincent doing when there is a vertical line on the graph?

this time is going down

b. Why are the vertical lines of different lengths?

because theres different data

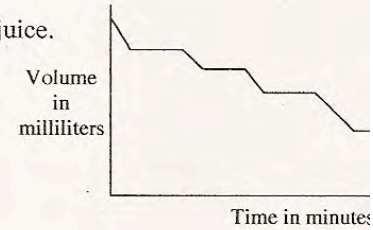
c. Did Vincent eat all the raisins? no ✓

Explain how you know.

because he only ate half of it

2. Ellie is drinking with a straw from a box of fruit juice.

The graph shows the volume of juice in the box as time passes.

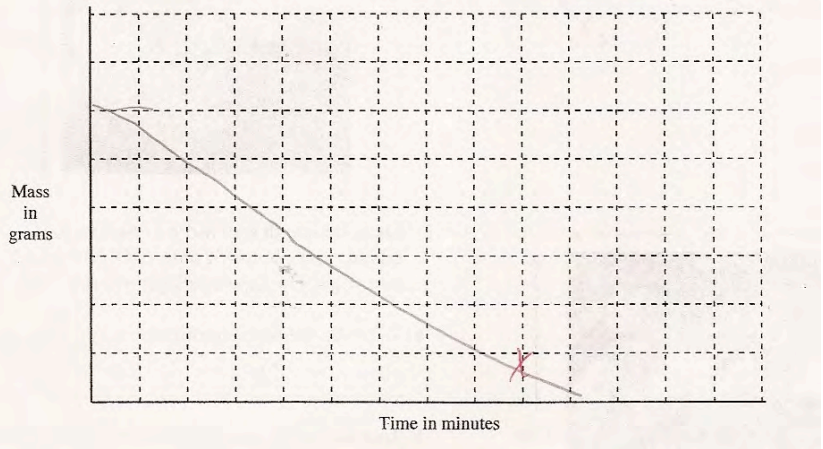


a. What is happening when the line on the graph is horizontal?

either it goes up or down

b. Why do the lines going downwards on this graph go at an angle?

because it goes down



Student Task	Draw and interpret graphs.
Core Idea 3 Algebra and Function	**Understand relations and functions, analyze mathematical situations, and use models to solve problems involving quantity and change.** • Explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope.
Core Idea 2 Mathematical Reasoning	**Employ forms of mathematical reasoning and justification appropriately to the solution of a problem.** • Verify and interpret results of a problem. • Use mathematical language and representations to make complex situations easier to understand.

Mathematics of the task:

- Make connections between the shape of the graph and the context
- Understand that adding or taking away objects on a scale causes an instantaneous change in weight
- Think about slope and the action of a story or context
- Make a graph to match a verbal description of an action, thinking about the slopes and the different discrete actions taking place

Based on teacher observations, this is what eighth graders know and are able to do:

- Understand the downward motion of removing cherries from a bag and graph that action
- Understand allowing time to pass before removing more cherries and graphing that action
- Explain how they know that Vincent did not eat all the raisins
- Understand the meaning of the horizontal line on Ellie's graph, drinking juice

Areas of difficulty for eighth graders:

- Understanding how to graph returning the cherry pit to the bag
- Distinguishing instantaneous weight change, removing raisins from the bag, from eating raisins
- Understanding that a long horizontal line does not mean that all the raisins are gone, just that no action is occurring
- Confusing labels on the graph with the action of the story; not relating graph to context

MARS Test Task 4 Frequency Distribution and Bar Graph, Grade 8

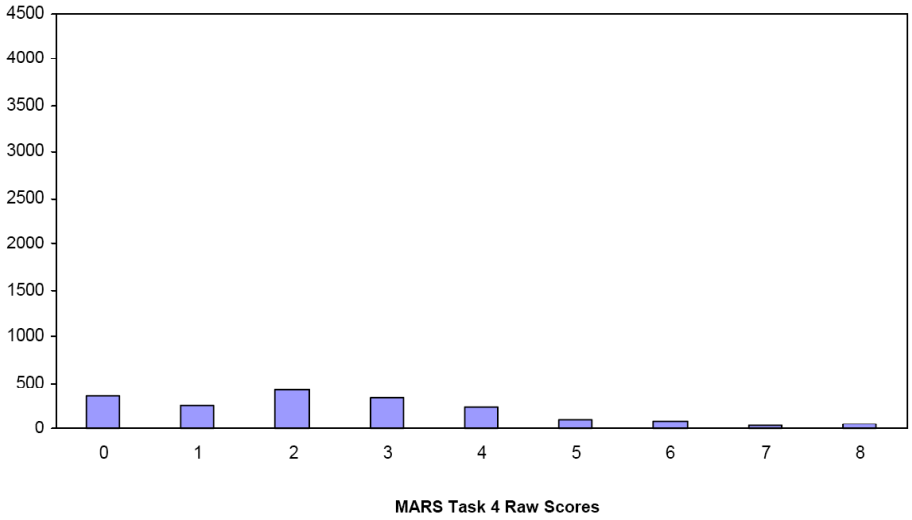
Task 4 - Vincent's Graphs

Mean: 2.49 StdDev: 1.96

Table 43: Frequency Distribution of MARS Test Task 4, Grade 8

Task 4 Scores	Student Count	% at or below	% at or above
0	351	18.8%	100.0%
1	254	32.4%	81.2%
2	428	55.3%	67.6%
3	340	73.5%	44.7%
4	232	85.9%	26.5%
5	109	91.7%	14.1%
6	77	95.8%	8.3%
7	27	97.3%	4.2%
8	51	100.0%	2.7%

Figure 52: Bar Graph of MARS Test Task 4 Raw Scores, Grade 8



The maximum score available for this task is 8 points.
The minimum score needed for a level 3 response, meeting standards, is 4 points.

Many students, 67%, could understand the downward motion of removing cherries from a bag and allow time to pass between removing cherries and the next action and graph those actions. Some students, about 27%, could explain why the graph shows Vincent did not eat all the raisins, explain the horizontal line on Ellie's graph in terms of the context and graph 2 of the actions about the cherries. About 14% could also explain the downward slanted lines on Ellie's graph as the slow action of sipping. About 3% could meet all the demands of the graph including recognizing the difference between removing objects from a box, instantaneous, and eating raisins, which doesn't effect the weight of the box. Almost 19% of the students scored no points on the task. 83% of the students with this score attempted the task.

Vincent's Graph

Points	Understandings	Misunderstandings
0	83% of the students with this score attempted the task.	Students did not know how to make a time graph. Some drew bar graphs, some made lines with an upward trend.
2	Students could graph the vertical and horizontal lines on the graph.	Students did not understand the "returning the pit" to the bag. 52% made just a downward staircase, with no upward parts.
4	Students could graph the vertical and horizontal lines on the graph. Students could talk about why Vincent did not eat all the raisins and knew that Ellie had stopped drinking when the graph showed a horizontal line.	9% thought Vincent had eaten all the raisins because the line was flat or stopped going down. 9% thought he had not eaten all the raisins because the line never stops. 18% of the students thought Ellie was drinking during the horizontal lines on the graph. 6% thought that was the speed of her drinking, faster or slower.
5	Students could graph 2 of the 3 parts of the graph, explain that Vincent ate all the raisins, and interpret both parts of Ellie's graph in the context of the story.	11% of the students did not attempt to explain the slanted lines on Ellie's graph. Many talked about scale rather than context, how much juice is left or drinking as time goes by. Many thought it had to do with the volume being consumed.
8	Students could interpret time graphs in terms of the action they represent and make a time graph of their own.	

Implication for Instruction

Students need more work with interpreting graphs in context. Students at this grade should develop an understanding of slope and the relationship to what activities are occurring over time. For example, the weight of items on a scale goes down instantaneously when items are removed.

Ideas for Action Research

Making Graphs for Stories

Having students make their own graphs about situations helps them to understand the logic of the graph and see how the lines do not represent the action of the story. Consider giving your students some story situations and have them make a graph of the general situation, not necessarily dealing with the issues of an exact scale. For example:

A factory cafeteria contains a vending machine selling drinks.
On a typical day:
- The machine starts half full.
- No drinks are sold before 9 a.m. or after 5 p.m.
- Drinks are sold at a slow rate throughout the day, except during the morning and lunch breads (10:30-11 am and 102 pm) when there is a greater demand.
- The machine is filled up just before the lunch break. (It takes about 10 minutes to fill).

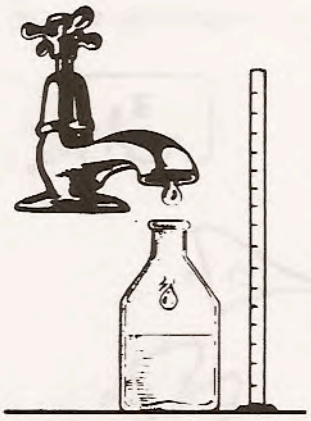
Make a sketch to the graph to show how the number of drinks in the machine might vary from 8 am to 6 pm.

What does the student have to understand about graphing to do this task? What do you think the graph might look like? What errors do you anticipate students might make? How does this help you think about how to process this activity? What are the mathematics you want to bring out or highlight as students discuss their work?

Now consider another task from the Shell Centre book, The Language of Functions and Graphs.

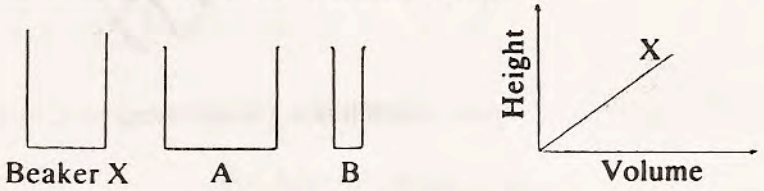
A5 LOOKING AT GRADIENTS

Filling Bottles

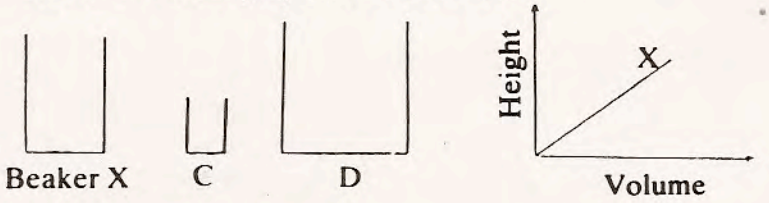


In order to callibrate a bottle so that it may be used to measure liquids, it is necessary to know how the height of the liquid depends upon the volume in the bottle.

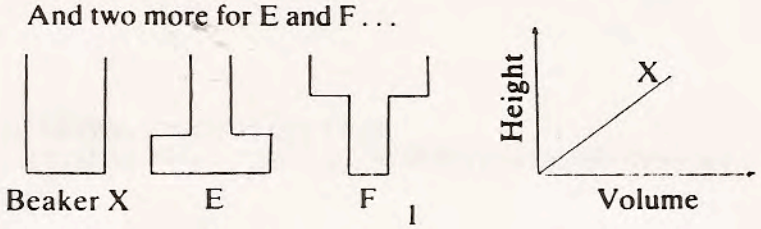
The graph below shows how the height of liquid in beaker X varies as water is steadily dripped into it. Copy the graph, and *on the same diagram,* show the height-volume relationship for beakers A and B.



Sketch two more graphs for C and D . . .



And two more for E and F . . .



Choose the best graph to fit each of the ten situations described below. (Particular graphs may fit more than one situation.) Copy the graph, label your axes and explain your choice, stating any assumptions you make. If you cannot find the graph you want, draw your own version.

1. "Prices are now rising more slowly than at any time during the last five years."
2. "I quite enjoy cold milk or hot milk, but I loathe lukewarm milk!"
3. "The smaller the boxes are, then the more boxes we can load into the van."
4. "After the concert there was a stunned silence. Then one person in the audience began to clap. Gradually, those around her joined in and soon everyone was applauding and cheering."
5. "If cinema admission charges are too low, then the owners will lose money. On the other hand, if they are too high then few people will attend and again they will lose. A cinema must therefore charge a moderate price in order to stay profitable."

19 (82)

In the following situations, *you* have to decide what happens. Explain them carefully in words, and choose the best graph, as before.

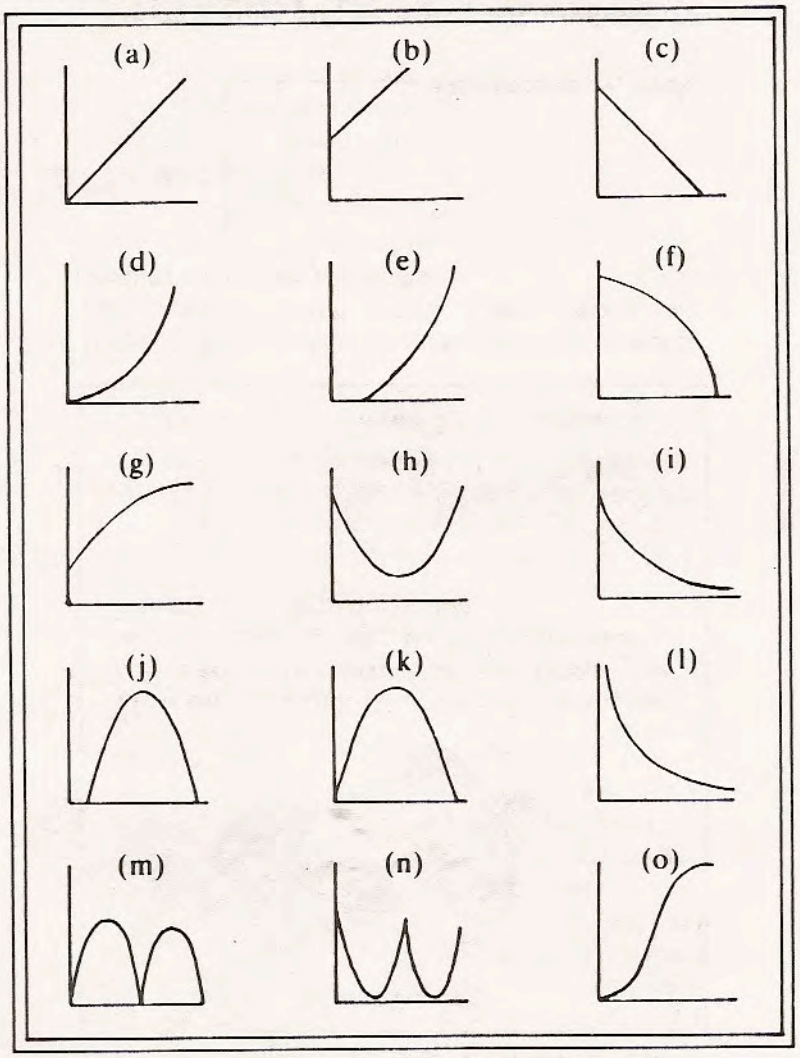
How does . . .

6. the cost of a bag of potatoes depend on its weight?
7. the diameter of a balloon vary as air is slowly released from it?
8. the time for running a race depend upon the length of the race?

' Shell Centre for Mathematical Education, University of Nottingham, 1985.

9. the speed of a girl vary on a swing?

10. the speed of a ball vary as it bounces along?



3

How did this activity add to student understanding of line graphs? What did the student have to think about to be successful? What evidence of understanding did you see in student work?

Performance Assessment Task

Vincent's Graphs
Grade 8

This task challenges a student to use understanding of functions to interpret and draw graphs. A student must be able to analyze a graph and understand the functional relationship. A student must be able to represent a mathematical situation and model using a graph. A student must be able to design or describe a real-life situation to fit a particular function.

Common Core State Standards Math - Content Standards

Functions

Use functions to model relationships between quantities.

8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (**x**, **y**) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Common Core State Standards Math – Standards of Mathematical Practice

MP.2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
8	2009	8	4	32%